Transmit Beamforming and Power Control for Optimizing the Outage Probability Fairness in MISO Networks

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***Abstract*— This paper studies the joint beamforming and power control in a multiuser multi-input single-output network by utilizing the only statistical channel distribution information. Such information consists of slowly varying covariance matrices in the beamforming network that can be employed to reduce instantaneous feedback overhead in transmission. Utilizing solely the statistical channel information, we study how to minimize the maximum outage probability under a weighted sum power constraint that guarantees max–min fairness to all users. This problem is, however, generally hard to solve due to the nonconvexity and nonlinear coupling between beamformer and power variables. First, assuming a fixed beamformer set, we use the nonlinear Perron–Frobenius theory to design a decentralized algorithm with provable geometrically fast convergence rate to compute the optimal power. Then, for the general case, we examine a certainty-equivalent margin counterpart with outage-mapped thresholds that incorporate the statistical channel information. We show that a network duality for this certainty-equivalent problem can be useful to decouple the coupling between the beamformer and power variables. This nonlinear Perron–Frobenius theory motivated approach yields a feasible beamformer and power allocation that are near-optimal as compared to Monte Carlo averaging simulations.**

***Index Terms*— Outage probability, power control, transmit beamforming, statistical channel information, nonlinear Perron-Frobenius theory, wireless networks.**

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I. Introduction

In multiuser wireless networks, power control helps in energy saving and interference reduction and thereby enables a cost-effective usage of the wireless resources [1]. In addition, with the deployment of antenna arrays at the transmitter side, efficient beamforming techniques can be employed jointly with power control to exploit spatial correlation in multiuser Multi-Input Single-Output (MISO) networks to improve the spectrum usage [2]–[6]. Most of the works on joint power control and beamforming in the literature (cf. [2]–[4]) assume the availability of instantaneous channel state information (CSI) of the wireless environment for transmission. To obtain the instantaneous CSI, each receiver has to estimate the channel and then sends the information by feedback to the transmitter. However, when there is a very large number of transmit-receive links and the instantaneous channel fluctuates too rapidly for tracking, this feedback requirement may become prohibitive and even practically infeasible in a network with a large number of users [7]. Indeed, next-generation wireless networks will have higher capacity, allowing a higher number of mobile broadband users per area unit. Thus, it is important to study efficient transmission strategies with either limited feedback, partial feedback, e.g., see [8] or even open-loop transmission without feedback.

In this paper, we study the use of statistical channel distribution information (CDI) for optimizing the statistical performance of multiuser transmission. Now, the CDI reflects the existing correlation information at the antenna side [9], and is known to be slowly varying (on the order of tens of seconds or more). Transmission strategies based on the CDI are triggered only when the statistical channel information has changed. Thus, CDI-based transmission strategies can have a lower implementation complexity and, in practice, are more stable and robust as compared with transmission schemes relying on the CSI [10]. However, in a CDI-based system, reliable transmission cannot be guaranteed at all time due to the statistical variation of the instantaneous channel. In other words, a transmission outage can occur when the received signal-to-interference-plus-noise ratio (SINR) is less than a pre-determined threshold that depends on the transmission quality-of-service requirement. Herein, we study the joint optimization of beamformer and power to minimize the maximum outage probability, i.e., provide a guarantee against the worst case in the statistical sense.

The seminal work that adopted the CDI approach to design power control (without multiple antennae beamforming) to optimize the outage probability fairness was studied in [11] for a restrictive interference-limited special case (zero noise power and without power constraints) using geometric programming. In [12] and [13], the authors extended the total power minimization problem to a Multi-Input Multi-Output (MIMO) network with antennas at both the transmit and receive sides, and designed iterative power control algorithms.

The total power minimization problem with outage specification constraints in a multiuser uplink was solved using an algorithm based on the standard interference function framework in [13] and [14]. The author in [15] and [16] studied the outage probability max-min fairness problem in [11] for the generalized case (positive noise power and power constraints) analytically using the nonlinear Perron-Frobenius theory in [17]–[20]. The nonlinear Perron-Frobenius theory approach transforms a nonconvex optimization problem into an equivalent conditional eigenvalue problem that can be solved optimally. Furthermore, it provides a systematic algorithm design to compute the optimal solution, and has been used to resolve the convergence problem of a heuristic algorithm proposed in [11].

In this paper, we study the outage probability max-min fairness problem with a weighted sum power constraint in a multiuser MISO network, where all the links experience correlated Rayleigh fading. This is a much harder problem due to the correlation coupling between users, and the nonconvexity in the power vector and beamforming weights. We first present the optimal solution assuming a fixed optimal set of beamformer. Since jointly optimizing the beamformer and power for the outage probability max-min fairness problem is nonconvex, we analyze its certainty-equivalent margin counterpart [11]–[13], which is an approximation to the CDI approach. By leveraging the nonlinear Perron-Frobenius theory [17]–[20], we propose an iterative algorithm to compute near-optimal beamforming weights. With this beamformer, we can obtain a feasible solution of the outage probability max-min fairness problem. The geometrically fast convergence rate of our algorithm is further proven. Lastly, we evaluate the performance of our CDI-based algorithm by comparing our approach to a baseline method using Monte-Carlo averaging simulations.

Overall, the contributions of the paper are as follows:

1) First, we characterize how to solve the outage probability max-min fairness problem with a generalized weighted sum power constraint by linking it to the solution of another problem that we call the certainty-equivalent margin problem with specially-constructed outage-mapped thresholds.
2) Second, we propose a decentralized algorithm to compute a feasible solution for the power and beamformer, and we show that this approximation to balance the outage probabilities among all users is near-optimal.
3) Third, we establish an uplink-downlink network duality for this certainty-equivalent margin problem using a geometric programming duality, and we evaluate our proposed algorithm by using Monte-Carlo averaging simulations.

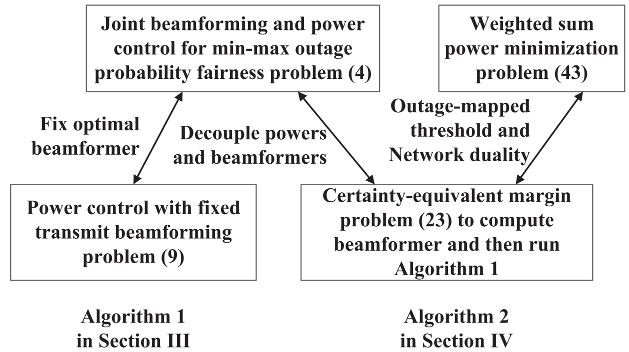

Fig. 1. Connection between the three key optimization problems considered in this paper and the algorithm design methodologies.

The paper is organized as follows. Section II presents the system model and the outage probability max-min fairness problem. In Section III, we examine the problem assuming the knowledge of the optimal beamformer. Section IV studies the case without the assumption, and analyzes the certainty-equivalent margin of the original problem. Section V provide the justification of the duality in our proposed algorithm. We evaluate the performance and compare the CDI-based algorithm with the CEM method with outage-mapped thresholds in Section VI. Finally, Section VII concludes the paper. Figure 1 presents an overview of the key optimization problems introduced in this paper and a summary of the solution methodologies.

Notations in this paper are presented as follows. Boldface upper-case letters denote matrices, boldface lower-case letters denote vectors, and italics denote scalars. The Perron-Frobenius eigenvalue of a nonnegative matrix $\mathbf{F}$ is denoted by $\rho(\mathbf{F})$. $\mathbf{x}(\mathbf{F})$ and $\mathbf{y}(\mathbf{F})$ denote the Perron-Frobenius right and left eigenvectors of $\mathbf{F}$ associated with $\rho(\mathbf{F})$ respectively. $\text{diag}(\mathbf{a})$ denotes the diagonal matrix having the vector $\mathbf{a}$ on its diagonal. We let $\mathbf{a} \circ \mathbf{b} \triangleq (a_1 b_1, \cdots, a_K b_K)^\top$, i.e., the Schur product. $\mathbb{C}$, $\mathbb{R}_+$, and $\mathbb{R}_{++}$ represent the set of complex numbers, the set of nonnegative real numbers, and the set of positive real numbers respectively. $(\cdot)^\top$ and $(\cdot)^\dagger$ denote the transpose operation and conjugate transpose operation respectively. $\|\cdot\|$ denotes the Euclidean norm of vectors.

II. System Model

Let us consider a multiuser MISO network with K transmit-receive pairs, as shown in Figure 2. Each transmitter is assumed to be equipped with N antennas. The received signal y_k for user (receiver) k is written as:

$$y_k = \mathbf{h}_{k,k}^\dagger \mathbf{x}_k + \sum_{j \neq k} \mathbf{h}_{k,j}^\dagger \mathbf{x}_j + z_k, \tag{1}$$

where $\mathbf{h}_{k,j} \in \mathbb{C}^{N \times 1}$ denotes the channel vector between transmitter j and user k, $\mathbf{x}_k \in \mathbb{C}^{N \times 1}$ is the transmit signal vector of transmitter k, and z_k characterizes the additive white noise effect, which is distributed as $\mathcal{CN}(0, \sigma_k)$ with $\sigma_k \in \mathbb{R}_{++}$.

Linear beamforming strategy is assumed at the transmitter side, thus the transmit signal vector $\mathbf{x}_k$ is expressed

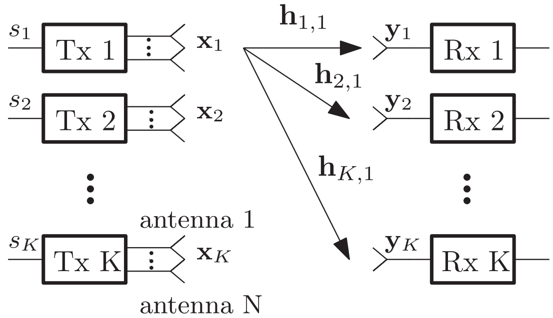

Fig. 2. A MISO downlink channel with K transmit-receive pairs. Each transmitter is equipped with N antennas. Linear beamforming and power control are performed at the transmitter.

as $\mathbf{x}_k = \sqrt{p_k} s_k \mathbf{u}_k$, where s_k and p_k denote the information signal and the transmit power for link k, and $\mathbf{u}_k \in \mathbb{C}^{N\times 1}$ denotes the normalized transmit beamformer for user k, i.e., $\|\mathbf{u}_k\|^2 = 1$. At time slot t, a channel vector realization $\mathbf{h}_{k,j}(t)$ is obtained independently for all k and j. In this paper, the correlated Rayleigh fading is assumed for all the links in the network, modeled as:

$$\mathbf{h}_{k,j}(t) \sim \mathcal{CN}(0, \Sigma_{k,j}), \tag{2}$$

where the Hermitian and positive semi-definite $\Sigma_{k,j}$ is the covariance matrix, i.e.:

$$\mathbb{E}[\mathbf{h}_{k,j}(t)\mathbf{h}_{k,j}^\dagger(t)] = \Sigma_{k,j}, \quad \forall k, j,$$

where $\mathbb{E}[\cdot]$ is the expectation operator in probability theory. Now, for a fixed t, $\mathbf{h}_{k,j}(t)\mathbf{h}_{k,j}^\dagger(t)$ has rank one while the rank of $\Sigma_{k,j}$ in fact takes value from one to N. This indicates a difference between the instantaneous CSI case and the CDI case which may, to some degree, determine the outage probability.

Under transmission with channel fading, the instantaneous SINR is in fact a random variable that depends on the instantaneous channel realization. In general, the SINR for user k is written as:

$$\mathsf{SINR}_k(\mathbf{p}, \mathbb{U}) = \frac{p_k|\mathbf{h}_{k,k}(t)^\dagger \mathbf{u}_k|^2}{\sum_{j\neq k} p_j|\mathbf{h}_{k,j}(t)^\dagger \mathbf{u}_j|^2 + \sigma_k}, \tag{3}$$

where $\mathbf{p} = (p_1, \cdots, p_K)^\mathsf{T}$, $\mathbb{U} = (\mathbf{u}_1, \cdots, \mathbf{u}_K)$ at a discrete time slot t.

Now, if the instantaneous CSI is available at the transmitter side of all the users, instantaneous adaptation of the beamformer and power is possible to optimize the SINR in (3) for each user. This approach however requires each user to actively track the channel state fluctuation over time and further leads to a large amount of feedback overhead. This is in general not practically implementable when the channel changes fast enough. Herein, the CDI which characterizes the channel covariance matrices is assumed to be available at the transmitter side, and is used to optimize the transmit power and beamformer. The advantage of a CDI-based approach is thus its low implementation complexity and suitability for large-scale networks with many users.

Let us denote the SINR minimum threshold for link k as β_k, then SINR_k can fall below β_k with some probability due to the channel fading. In other words, the transmission strategies based only on the CDI can experience a fading-induced outage event at each time slot. The outage probability for link k is expressed as $\mathbb{P}(\mathsf{SINR}_k(\mathbf{p}, \mathbb{U}) < \beta_k)$. From a system design perspective, it is desired that the worst case of the outage probability, i.e., the largest outage probability among all the users, can be made as small as possible in order to guarantee a max-min fairness to all the users. Furthermore, linear power constraints appear in many kinds of ad-hoc settings, e.g., interference temperature constraints in wireless cognitive radio settings. In this paper, we focus on a linear power constraint that is applicable to the cellular downlink networks. For ease of exposition and without loss of the generality, let w_k denote the weight associated with p_k for user k with $\mathbf{w} = (w_1, \ldots, w_K)^\mathsf{T}$ which is used to model different power prices for different users, and all the users are subject to a single weighted sum power constraint.

The outage probability max-min fairness problem subject to a weighted sum power constraint is formulated as follows:

$$\begin{aligned} &\text{minimize } \max_k \ \mathbb{P}(\mathsf{SINR}_k(\mathbf{p}, \mathbb{U}) < \beta_k) \\ &\text{subject to } \mathbf{w}^\mathsf{T}\mathbf{p} \le \bar{P}, \\ &\qquad\qquad p_k \ge 0, \quad \|\mathbf{u}_k\|^2 = 1 \quad \forall k, \\ &\text{variables : } \mathbf{p}, \mathbb{U}. \end{aligned} \tag{4}$$

Now, (4) is generally hard to solve due to its nonconvexity and the tight coupling between the power and beamformer variables. Special cases when the beamformers are fixed have been studied in [16], [19], and [20]. In the following, we will propose analytical results and algorithms to solve (4). Let us also denote the threshold vector $\boldsymbol{\beta} = (\beta_1, \cdots, \beta_K)^\mathsf{T}$ and the noise power vector $\boldsymbol{\sigma} = (\sigma_1, \cdots, \sigma_K)^\mathsf{T}$.

III. Analysis and Algorithm Design

A. Problem Formulation

In order to further analyze the optimization problem in (4), an explicit deterministic expression for the outage probability can be helpful to transform (4) from a stochastic problem to a static optimization problem. Using the results developed in [11], [12], and [21], a closed form expression for $\mathbb{P}(\mathsf{SINR}_k(\mathbf{p}, \mathbb{U}) < \beta_k)$ can be obtained as follows.

Lemma 1: In a multiuser MISO network where all the links experience the correlated Rayleigh fading, the closed form expression of the outage probability for link k is written as[1]:

$$\mathbb{P}(\mathsf{SINR}_k(\mathbf{p}, \mathbb{U}) < \beta_k) = 1 - e^{-\frac{\beta_k \sigma_k}{p_k c_{k,k}}} \prod_{j\neq k}\left(1 + \frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right)^{-1}, \tag{5}$$

where $c_{k,j}$ represents the statistical beamforming channel gain in terms of the beamforming matrix $\mathbb{U}$, whose expression is given by:

$$c_{k,j} \triangleq \mathbf{u}_j^\dagger \Sigma_{k,j} \mathbf{u}_j. \tag{6}$$

[1]This lemma corrects an error in [12, Th. 1] where a constant 2 should not appear in $e^{-\frac{\beta_k \sigma_k}{p_k c_{k,k}}}$.

Remark 1: The expression in (5) consists of the product of two major parts, i.e., $\prod_{j\neq k}\left(1+\frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right)^{-1}$ and $e^{-\frac{\beta_k\sigma_k}{p_k c_{k,k}}}$. The first part is due to the effect of multiuser interference from the other users, while the second part is due solely to the additive white Gaussian noise.

Using the result of Lemma 1, (4) is transformed to a deterministic optimization problem as follows:

$$\begin{aligned}&\text{minimize } \max_k \ 1-e^{-\frac{\beta_k\sigma_k}{p_k c_{k,k}}}\prod_{j\neq k}\left(1+\frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right)^{-1}\\ &\text{subject to } \mathbf{w}^{\mathsf{T}}\mathbf{p}\leq \bar{P},\\ &\qquad p_k>0,\quad \|\mathbf{u}_k\|^2=1\ \forall k,\\ &\text{variables}: \mathbf{p},\mathbb{U}.\end{aligned}\tag{7}$$

Next, we introduce the auxiliary variable τ to transform (7) into the epigraph formulation as:

$$\begin{aligned}&\text{minimize } \tau\\ &\text{subject to } \frac{\beta_k\sigma_k}{p_k c_{k,k}}+\sum_{j\neq k}\log\left(1+\frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right)\leq\tau\ \forall k,\\ &\qquad \mathbf{w}^{\mathsf{T}}\mathbf{p}\leq\bar{P},\quad p_k>0,\quad \|\mathbf{u}_k\|^2=1\ \forall k,\\ &\text{variables}: \mathbf{p},\mathbb{U},\tau.\end{aligned}\tag{8}$$

Although the problem (8) is nonconvex in $(\mathbf{p},\mathbb{U},\tau)$, the optimal $\mathbf{p}^\star$ can be given in terms of the beamformer $\mathbf{p}^\star(\mathbb{U}^\star)$ by making the assumption that the optimal beamformer solution is known and given. In the next part, we will leverage the results in [16] and [19] to derive $(\mathbf{p}^\star(\mathbb{U}),\tau^\star(\mathbb{U}))$ using this assumption to draw insights before giving a general solution to solve (8) without this knowledge of the optimal beamformer.

B. Optimal Solution Under Fixed Beamformer

Suppose that we are given the optimal set of beamformer $\mathbb{U}^\star$, then a simpler optimization problem is obtained from (8) as follows:

$$\begin{aligned}&\text{minimize } \tau(\mathbb{U})\\ &\text{subject to } \sum_{j\neq k}\log\left(1+\frac{\beta_k p_j(\mathbb{U})c_{k,j}(\mathbb{U})}{p_k(\mathbb{U})c_{k,k}(\mathbb{U})}\right)\\ &\qquad +\frac{\beta_k\sigma_k}{p_k(\mathbb{U})c_{k,k}(\mathbb{U})}\leq\tau(\mathbb{U})\ \forall k,\\ &\qquad \mathbf{w}^{\mathsf{T}}\mathbf{p}(\mathbb{U})\leq\bar{P},\quad p_k(\mathbb{U})>0\ \forall k,\\ &\text{variables}: \mathbf{p}(\mathbb{U}),\tau(\mathbb{U}).\end{aligned}\tag{9}$$

Let us denote the first K constraints of (9) as outage constraints. Following [16] and [19], we examine (9) using the nonlinear Perron-Frobenius theory to derive a fixed-point problem.

Lemma 2: The outage constraints as well as the weighted sum power constraint become active at optimality, i.e.:

$$\begin{cases}\dfrac{\beta_k\sigma_k}{p_k(\mathbb{U})c_{k,k}(\mathbb{U})}+\sum_{j\neq k}\log\left(1+\dfrac{\beta_k p_j(\mathbb{U})c_{k,j}(\mathbb{U})}{p_k(\mathbb{U})c_{k,k}(\mathbb{U})}\right)\\ \qquad\qquad =\tau(\mathbb{U}),\ \forall k,\\ \mathbf{w}^{\mathsf{T}}\mathbf{p}(\mathbb{U})=\bar{P}.\end{cases}\tag{10}$$

Proof: It is easy to verify that the left-hand side of the k-th outage constraint is monotonically increasing in $p_j(\mathbb{U})$ for $j\neq k$ and monotonically decreasing in $p_k(\mathbb{U})$.

Suppose that there is an outage constraint of a specific user k that is not tight at optimality, i.e.,

$$\begin{aligned}&\frac{\beta_k\sigma_k}{p_k^\star(\mathbb{U}^\star)c_{k,k}(\mathbb{U}^\star)}+\sum_{j\neq k}\log\left(1+\frac{\beta_k p_j^\star(\mathbb{U}^\star)c_{k,j}(\mathbb{U}^\star)}{p_k^\star(\mathbb{U}^\star)c_{k,k}(\mathbb{U}^\star)}\right)\\ &\quad<\tau^\star(\mathbb{U}^\star).\end{aligned}\tag{11}$$

We can reduce the power p_k by a sufficiently small amount $\epsilon>0$ to obtain $\tau_k<\tau^\star$ so that (11) is still satisfied using a new transmit power for user k, i.e., $\hat{p}_k=p_k-\epsilon$. By doing so, the other users' outage probabilities decrease and their outage constraint requirements can still be satisfied. Meanwhile, the weighted sum power constraint can still be satisfied. Thus, we can further reduce the outage probability which is a contradiction to the assumption that the powers are optimal. Therefore, all the outage constraints become active at optimality.

Next, suppose that the weighted sum power constraint is not tight at optimality, i.e.:

$$\mathbf{w}^{\mathsf{T}}\mathbf{p}^\star(\mathbb{U}^\star)<\bar{P}.\tag{12}$$

We can increase all the power proportionally by a sufficiently small amount $\epsilon>0$ where $\tilde{p}_j=(1+\epsilon)p_j^\star$ for all users such that the weighted sum power constraint is still satisfied:

$$\mathbf{w}^{\mathsf{T}}\tilde{\mathbf{p}}=\mathbf{w}^{\mathsf{T}}\tilde{\mathbf{p}}^\star(\mathbb{U}^\star)+\epsilon\mathbf{w}^{\mathsf{T}}\tilde{\mathbf{p}}^\star(\mathbb{U}^\star)<\bar{P}.\tag{13}$$

Then, we get:

$$\log\left(1+\frac{\beta_k\tilde{p}_j c_{k,j}(\mathbb{U}^\star)}{\tilde{p}_k c_{k,k}(\mathbb{U}^\star)}\right)=\log\left(1+\frac{\beta_k p_j^\star(\mathbb{U}^\star)c_{k,j}(\mathbb{U}^\star)}{p_k^\star(\mathbb{U}^\star)c_{k,k}(\mathbb{U}^\star)}\right).\tag{14}$$

Thus, we can further reduce all the outage probabilities due to the increase of each transmit power, which is a contradiction to the assumption that the powers are optimal. Therefore,

$$\tau^\star\mathbf{p}^\star=\begin{bmatrix}\tau^\star p_1^\star\\ \vdots\\ \tau^\star p_K^\star\end{bmatrix}=\begin{bmatrix}\dfrac{\beta_1\sigma_1}{c_{1,1}}\\ \vdots\\ \dfrac{\beta_K\sigma_K}{c_{K,K}}\end{bmatrix}\times\frac{\mathbf{w}^\top\mathbf{p}^\star}{\bar{P}}+\begin{bmatrix}0 & \cdots & \dfrac{p_1^\star}{p_K^\star}\log\left(1+\dfrac{\beta_1 p_K^\star c_{1,K}}{p_1^\star c_{1,1}}\right)\\ \vdots & \ddots & \vdots\\ \dfrac{p_K^\star}{p_1^\star}\log\left(1+\dfrac{\beta_K p_1^\star c_{K,1}}{p_K^\star c_{K,K}}\right) & \cdots & 0\end{bmatrix}\mathbf{p}^\star.\tag{15}$$

Algorithm 1 Optimal Power Computation

- For a given $\mathbb{U}$, initialize arbitrary $\mathbf{p}[0] \in \mathbb{R}_{++}^{K\times 1}$ such that $\mathbf{w}^{\mathsf{T}}\mathbf{p}[0] \leq \bar{P}$.

1) Update power $\mathbf{p}[\ell+1]$ for all k:

$$p_k[\ell+1] = \frac{\beta_k\sigma_k}{c_{k,k}(\mathbb{U})} + \sum_{j\neq k} p_k[\ell] \log\left(1 + \frac{\beta_k p_j[\ell] c_{k,j}(\mathbb{U})}{p_k[\ell] c_{k,k}(\mathbb{U})}\right).$$

2) Normalize $\mathbf{p}[\ell+1]$:

$$\mathbf{p}[\ell+1] \leftarrow \frac{\bar{P}}{\mathbf{w}^{\mathsf{T}}\mathbf{p}[\ell+1]}\mathbf{p}[\ell+1].$$

the weighted sum power constraint becomes active at optimality. ■

Using Lemma 2, the constrained fixed-point problem in (10) can be transformed to the following optimality conditions (15), as shown at the bottom of the previous page.

For a compact representation, we define the nonnegative matrix $\Psi(\mathbf{p}) \in \mathbb{R}_{+}^{K\times K}$ as follows:

$$\Psi_{k,j}(\mathbf{p}) = \begin{cases} 0, & \text{if } k = j \\ \dfrac{p_k c_{k,k}}{\beta_k p_j} \log\left(1 + \dfrac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right), & \text{if } k \neq j. \end{cases} \tag{16}$$

We also define the auxiliary vector $\mathbf{g} \triangleq \left(\frac{1}{c_{1,1}}, \cdots, \frac{1}{c_{K,K}}\right)^{\mathsf{T}}$. Then, the optimal power vector satisfies the following conditional eigenvalue problem based on [6], [22]:

$$\tau^\star(\mathbb{U})\mathbf{p}^\star(\mathbb{U}) = \mathrm{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U}))\left(\frac{1}{\bar{P}}\boldsymbol{\sigma}\mathbf{w}^{\mathsf{T}} + \Psi(\mathbf{p}^\star(\mathbb{U}))\right)\mathbf{p}^\star(\mathbb{U}). \tag{17}$$

From (17), it is seen that according to the nonnegative matrix theory that $\mathbf{p}^\star(\mathbb{U})$ is the right Perron-Frobenius eigenvector (up to a scaling factor) of the nonnegative matrix $\mathrm{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U}))\left(\Psi(\mathbf{p}^\star(\mathbb{U})) + (1/\bar{P})\boldsymbol{\sigma}\mathbf{w}^{\mathsf{T}}\right)$, and $\tau^\star(\mathbb{U})$ is related to its spectral radius by the following:

$$\tau^\star(\mathbb{U}) = \rho\left(\mathrm{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U}))\left((1/\bar{P})\boldsymbol{\sigma}\mathbf{w}^{\mathsf{T}} + \Psi(\mathbf{p}^\star(\mathbb{U}))\right)\right). \tag{18}$$

In order to derive a fast algorithm to compute the optimal solution $\mathbf{p}^\star(\mathbb{U})$ and operate in a decentralized manner, we employ the nonlinear Perron-Frobenius theory to present Algorithm 1 as follows:

The geometrically fast convergence rate of Algorithm 1 is presented in the following theorem.

Theorem 1: Define the norm $\|\cdot\|_{\mathsf{PN}}$ on $\mathbb{R}_{+}^{K\times 1}$ as $\|\mathbf{p}\|_{\mathsf{PN}} = (1/\bar{P})\sum_k w_k|p_k|$, and the mapping $f^{(1)} : \mathbb{R}_{+}^{K\times 1} \to \mathbb{R}_{+}^{K\times 1}$ as

$$f_k^{(1)}(\mathbf{p}, \mathbb{U}) = \frac{\beta_k\sigma_k}{c_{k,k}(\mathbb{U})} + \sum_{j\neq k} p_k \log\left(1 + \frac{\beta_k p_j c_{k,j}(\mathbb{U})}{p_k c_{k,k}(\mathbb{U})}\right), \quad \forall k. \tag{19}$$

Then, the normalized fixed-point iteration $\hat{f}_k^{(1)}(\mathbf{p}[\ell+1], \mathbb{U}) = (1/\|f_k^{(1)}(\mathbf{p}[\ell], \mathbb{U})\|_{\mathsf{PN}}) f_k^{(1)}(\mathbf{p}[\ell], \mathbb{U})$ for any given $\mathbb{U}$ converges to the optimal solution of (9), i.e., $\mathbf{p}^\star(\mathbb{U})$, geometrically fast.

Proof: (*Sketch*) Following the similar technique as in [15] and [16], we can prove that $f^{(1)}(\mathbf{p}, \mathbb{U})$ is a concave self-mapping of $\mathbf{p}$ given $\mathbb{U}$. We also have $f_k^{(1)}(\mathbf{p}, \mathbb{U}) > 0$ for $\mathbf{p} > \mathbf{0}$. Then, the convergence property of the fixed-point iteration follows from [17, Th. 1] and [23]. ■

Remark 2: The power update in Algorithm 1 is distributed based on the given transmit beamformer $\mathbb{U}$. The normalization at Step 2 can be made distributed using gossip algorithms [24]–[26] to compute $\mathbf{w}^{\mathsf{T}}\mathbf{p}[\ell+1]$ at each user.

We have focused only on the optimal power given a fixed $\mathbb{U}$. However, due to the coupling between beamformers and powers in the outage constraints, it is difficult (if not impossible) to minimize $f_k^{(1)}(\mathbf{p}^\star(\mathbb{U}), \mathbb{U})$ to obtain the optimal beamformer. Since the outage probability max-min fairness problem (7) is nonconvex in $(\mathbf{p}, \mathbb{U})$, finding the optimal beamformer is still an open problem, and we provide an approach based on network duality in the next section.

IV. Approximation and Network Duality

A. Certainty-equivalent Margin Problem (CEM)

We consider the joint optimization problem (8) without the knowledge of the optimal beamformer $\mathbb{U}^\star$. To overcome the coupling, we consider a deterministic approximation technique used in [11] based on the so-called certainty-equivalent margin (CEM) counterpart of the original problem. This approach has also been employed in [12] and [13] for power minimization problems. We shall leverage this approach in our outage probability max-min fairness problem to derive a near-optimal solution.

Following [11], the upper and lower bounds for the left-hand side of the k-th outage constraint are derived as:

$$\begin{aligned} \log\left(1 + \frac{\beta_k\left(\sum_{j\neq k} p_j c_{k,j} + \sigma_k\right)}{p_k c_{k,k}}\right) &\leq \frac{\beta_k\sigma_k}{p_k c_{k,k}} + \sum_{j\neq k}\log\left(1 + \frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right) \\ &\leq \frac{\beta_k\left(\sum_{j\neq k} p_j c_{k,j} + \sigma_k\right)}{p_k c_{k,k}}. \end{aligned} \tag{20}$$

Now, the upper bound in (20) will be used to construct a new constraint to the original problem in (8) by considering:

$$\frac{\beta_k\left(\sum_{j\neq k} p_j c_{k,j} + \sigma_k\right)}{p_k c_{k,k}} \leq \tau, \tag{21}$$

where τ is a feasible solution in (8), i.e., $1 - e^{-\tau}$ is a feasible solution of (4). Interestingly, (21) can be viewed as a constraint on a function similar to a deterministic SINR-like expression with a specially-constructed threshold β_k/τ (instead of the original β_k) for all k. In particular, this leads us to formulate a certainty-equivalent margin problem, and we define $\Gamma_k^{\mathsf{PN}}(\mathbf{p}, \mathbb{U})$ as:

$$\Gamma_k^{\mathsf{PN}}(\mathbf{p}, \mathbb{U}) = \frac{p_k c_{k,k}}{\sum_{j \neq k} p_j c_{k,j} + \sigma_k}, \tag{22}$$

where the superscript $(\cdot)^{\mathsf{PN}}$ indicates the primal network. It can be easily seen that $\Gamma_k^{\mathsf{PN}}(\mathbf{p}, \mathbb{U})$ denotes the certainty-equivalent SINR-like expression as in [11] when the statistical variations of the signal as well as the interference are replaced by their expected values, and then imposing an outage-mapped threshold given by β_k/τ [13]. Then, we consider the following certainty-equivalent margin problem that has a specially-constructed weight as follows:

$$\begin{aligned} &\text{maximize} \ \min_k \ \frac{\Gamma_k^{\mathsf{PN}}(\mathbf{p}, \mathbb{U})}{\beta_k/\tau} \\ &\text{subject to } \mathbf{w}^\top \mathbf{p} \leq \bar{P}, \quad \mathbf{p} > 0, \ \ \|\mathbf{u}_k\|^2 = 1 \ \forall k, \\ &\text{variables}: \mathbf{p}, \mathbb{U}, \tau. \end{aligned} \tag{23}$$

We denote the optimal solution and optimal value of (23) as $(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}}, \tau^{\text{cem}})$ and $\zeta^\star$, respectively. We emphasize that, due to (21), this optimal solution is also feasible to the original problem in (4).

Next, we use the bounds (20) to explicitly express the relationship between the original outage probability max-min fairness problem (7) and its certainty-equivalent margin counterpart (23). From the last inequality in (20), we have:

$$\begin{aligned} O &= \max_k \left[1 - e^{-\frac{\beta_k \sigma_k}{p_k c_{k,k}}} \prod_{j \neq k} \left(1 + \frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}} \right)^{-1} \right] \\ &\leq \max_k \left(1 - e^{-\frac{\beta_k \sigma_k}{p_k c_{k,k}}} \cdot e^{-\sum \frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}} \right) \\ &= 1 - \exp\left(-\frac{1}{\min_k \frac{p_k c_{k,k}}{\beta_k (\sum_{j \neq k} p_j c_{k,j} + \sigma_k)}} \right) \\ &= 1 - e^{-\frac{\tau}{\zeta}}. \end{aligned} \tag{24}$$

By substituting the optimal solution of (23), $(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}}, \tau^{cem})$, into the above inequality (24), we get:

$$O(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}}) \leq 1 - e^{-\frac{\tau(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}})}{\zeta(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}})}} = 1 - e^{-\frac{\tau^{\text{cem}}}{\zeta^\star}}. \tag{25}$$

Since $O^\star = \min_{\mathbf{p}, \mathbb{U}} O$ and $(\mathbf{p}^\star, \mathbb{U}^\star)$ minimizes O, we can further conclude that:

$$O(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}}) \geq O(\mathbf{p}^\star(\mathbb{U}^{\text{cem}}), \mathbb{U}^{\text{cem}}) \geq O(\mathbf{p}^\star, \mathbb{U}^\star) = O^\star. \tag{26}$$

In a similar way, we get $O \geq \frac{\tau}{\tau + \zeta}$ and

$$\begin{aligned} O^\star &= O(\mathbf{p}^\star, \mathbb{U}^\star) \geq \frac{\tau(\mathbf{p}^\star, \mathbb{U}^\star)}{\tau(\mathbf{p}^\star, \mathbb{U}^\star) + \zeta(\mathbf{p}^\star, \mathbb{U}^\star)} \\ &\geq \frac{\tau(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}})}{\tau(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}}) + \zeta(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}})} = \frac{\tau^{\text{cem}}}{\tau^{\text{cem}} + \zeta^\star}. \end{aligned} \tag{27}$$

Combining the above three inequalities, we can give upper and lower bounds for $O^\star$ as follows:

$$\frac{\tau^{\text{cem}}}{\zeta^\star + \tau^{\text{cem}}} \leq O^\star \leq 1 - e^{-\frac{\tau^{\text{cem}}}{\zeta^\star}}. \tag{28}$$

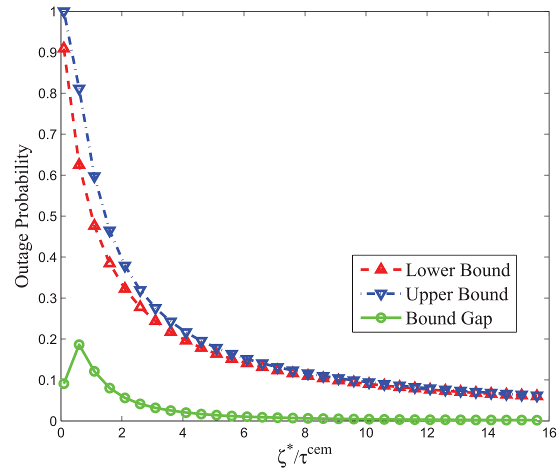

Fig. 3. The outage probabilities of the lower and upper bounds.

This near-optimal effect is observed in [11], [12], [15], and [16] as in Figure 3.

Moreover, it is known that $\log(1 + x) \approx x$ for small x, thus the upper and lower bounds become tighter for smaller outage probabilities. Therefore, the inequalities (26) and (28) reveal that the beamformer $\mathbb{U}^{\text{cem}}$ obtained by (23) can offer a useful $\mathbb{U}$ for Algorithm 1. In other words, $(\mathbf{p}^\star(\mathbb{U}^{\text{cem}}), \mathbb{U}^{\text{cem}})$ can be a near-optimal solution for the original problem (7). In the following, we first use the nonlinear Perron-Frobenius theory [18], [19], [22], [27] to examine (23) and then provide a fast iterative algorithm combined with Algorithm 1 to compute this solution.

B. Network Duality

For any beamformer $\mathbb{U}$, a simpler optimization problem for (23) can be formulated by only optimizing the power vector. It is shown in Lemma 2 that at optimality, the weighted power constraint becomes tight, and the weighted certainty-equivalent SINR for different users are the same. We define the nonnegative matrix $\mathbf{C} \in \mathbb{R}_+^{K \times K}$ as:

$$C_{k,j} = \begin{cases} 0, & \text{if } k = j \\ c_{k,j}, & \text{if } k \neq j. \end{cases} \tag{29}$$

We can interpret $\mathbf{C}$ as the average cross channel interference matrix. With the definition of $\mathbf{C}$, the optimal power vector satisfies the following eigenvalue problem:

$$\frac{\mathbf{p}^{\text{cem}}(\mathbb{U})}{\zeta^\star(\mathbb{U})} = \frac{\text{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U})) \left(\mathbf{C}(\mathbb{U}) + (1/\bar{P}) \boldsymbol{\sigma} \mathbf{w}^\top \right) \mathbf{p}^{\text{cem}}(\mathbb{U})}{\tau^{\text{cem}}(\mathbb{U})}. \tag{30}$$

Therefore, $\mathbf{p}^{\text{cem}}(\mathbb{U})$ is the right Perron-Frobenius eigenvector (up to a scaling factor) of the nonnegative matrix $\text{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U})) \left(\mathbf{C}(\mathbb{U}) + (1/\bar{P}) \boldsymbol{\sigma} \mathbf{w}^\top \right)$, and $\zeta^\star(\mathbb{U})$ is related to its spectral radius as follows:

$$\zeta^\star(\mathbb{U}) = \frac{\tau^{\text{cem}}(\mathbb{U})}{\rho \left(\text{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U})) \left(\mathbf{C}(\mathbb{U}) + (1/\bar{P}) \boldsymbol{\sigma} \mathbf{w}^\top \right) \right)}. \tag{31}$$

Now, we establish the hypothesized dual network for further analysis [3], [28], [29]. Denote the dual network transmit power vector as $\mathbf{q} \in \mathbb{R}_{++}^{K\times 1}$. Let the weight vector $\mathbf{w}$ in the primal network be the noise vector in the dual network, and conversely let the noise vector $\boldsymbol{\sigma}$ in the primal network be the weight vector in the dual network. Given a receive beamformer $\mathbb{U}$, the optimization for the dual network is formulated as:

$$\begin{aligned} &\text{maximize } \min_k \frac{\Gamma_k^{\mathsf{DN}}(\mathbf{q}, \mathbb{U})}{\beta_k/\tau} \\ &= \frac{\tau(\mathbb{U}) q_k(\mathbb{U})}{\left(\operatorname{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U}))\left(\mathbf{C}^{\mathsf{T}}(\mathbb{U})\mathbf{q}(\mathbb{U}) + \mathbf{w}\right)\right)_k} \\ &\text{subject to } \boldsymbol{\sigma}^{\mathsf{T}}\mathbf{q}(\mathbb{U}) \le \bar{P}, \quad \mathbf{q}(\mathbb{U}) > 0, \ \ \|\mathbf{u}_k\|^2 = 1 \ \forall k, \\ &\text{variables}: \mathbf{q}, \mathbb{U}, \tau, \end{aligned} \tag{32}$$

where the superscript $(\cdot)^{\mathsf{DN}}$ indicates the dual uplink network. By leveraging the properties of nonnegative matrices that $\rho(\mathbf{A}) = \rho(\mathbf{A}^{\mathsf{T}})$ and $\rho(\mathbf{AB}) = \rho(\mathbf{BA})$, the optimal solution of (32) equals $\tau(\mathbb{U})\rho^{-1}(\operatorname{diag}(\boldsymbol{\beta} \circ \mathbf{g}(\mathbb{U}))(\mathbf{C}^{\mathsf{T}}(\mathbb{U}) + (1/\bar{P})\mathbf{w}\boldsymbol{\sigma}^{\mathsf{T}}))$ [18], [19], [22], [27].

Comparing it with the optimal solution for the primal network in (30), the network duality is observed by employing $\mathbf{C}^{\mathsf{T}}$ as the average cross channel interference matrix for the dual network and reversing the role of $\mathbf{w}$ and $\boldsymbol{\sigma}$. Note that the network duality holds for any given $\mathbb{U}$. We give an analytical justification of this uplink-downlink duality using a geometric programming duality derivation in Section IV later.

The benefit of the established network duality is the decoupled property of the dual network which enables beamformer optimization. The optimal beamformer $\mathbb{U}^{\text{cem}}$ depends on the power vector $\mathbf{q}$ of the hypothesized dual network. For any given $\mathbf{q}$, the optimal beamformer $\mathbf{u}_k^{\text{cem}}(\mathbf{q})$ for link k is determined by the following equation:

$$\mathbf{u}_k^{\text{cem}}(\mathbf{q}) = \arg\max_{\mathbf{u}_k(\mathbf{q})} \frac{\mathbf{u}_k^{\dagger}(\mathbf{q})\Sigma_{k,k}\mathbf{u}_k(\mathbf{q})}{\mathbf{u}_k^{\dagger}(\mathbf{q})(\sum_{j\neq k} q_j \Sigma_{j,k} + w_k \mathbf{I})\mathbf{u}_k(\mathbf{q})}. \tag{33}$$

Therefore, the optimal beamformer is the dominant eigenvector of the generalized eigenvalue problem, which is well studied in [22] and [30]. In particular, $\mathbf{u}_k^{\text{cem}}(\mathbf{q})$ is the normalized vector satisfying the following equation with the largest λ:

$$\Sigma_{k,k}\mathbf{z} = \lambda \left(\sum_{j\neq k} q_j \Sigma_{j,k} + w_k \mathbf{I} \right) \mathbf{z}. \tag{34}$$

The optimal solution for the beamformer, the power of the dual network and the primal network, the outage-mapped threshold and the optimal value for (23) can be written as $\mathbf{u}_k^{\text{cem}} = \mathbf{u}_k^{\text{cem}}(\mathbf{q}^{\text{cem}})$, $\mathbf{q}^{\text{cem}} = \mathbf{q}^{\text{cem}}(\mathbb{U}^{\text{cem}})$, $\mathbf{p}^{\text{cem}} = \mathbf{p}^{\text{cem}}(\mathbb{U}^{\text{cem}})$, $\tau^{\text{cem}} = \tau^{\text{cem}}(\mathbb{U}^{\text{cem}})$ and $\zeta^{\star} = \zeta^{\star}(\mathbb{U}^{\text{cem}})$ respectively. As mentioned before, $\zeta^{\star}/\tau^{\text{cem}}$ can be used to bound the optimal outage probability. We note that what we need in the problem (23) is $\mathbb{U}^{\text{cem}}$, while $\mathbf{p}^{\text{cem}}$ and $\mathbf{q}^{\text{cem}}$ are just by-products that assist in the computation of $\mathbb{U}^{\text{cem}}$. In the next part, we present an iterative algorithm to compute the near-optimal solution of (7) and also discuss the convergence behavior and its complexity.

Algorithm 2 Joint Power Control and Beamformer Computation

- Initialize arbitrary power in (23) and beamformer, respectively: $\mathbf{m}[0] \in \mathbb{R}_{++}^{K\times 1}$, $\mathbf{n}[0] \in \mathbb{R}_{++}^{K\times 1}$ and $\mathbf{u}_k[0] \in \mathbb{C}^{N\times 1}$ for $k = 1, \dots, K$ such that $\|\mathbf{u}_k[0]\| = 1, \forall k$, $\mathbf{w}^{\mathsf{T}}\mathbf{m}[0] \le \bar{P}$, and $\boldsymbol{\sigma}^{\mathsf{T}}\mathbf{n}[0] \le \bar{P}$. Here we denote the power expression in (23) as $\mathbf{m}$ and $\mathbf{n}$ instead of $\mathbf{p}$ and $\mathbf{q}$ to distinguish these auxiliary variables from the "real power" in (9) for actual transmission, which is shown in Step 8.

1) Update the outage-mapped threshold:
$$\tau[\ell] = \min_k \left(\sum_{j\neq k} \log\left(1 + \frac{\beta_k m_j[\ell] c_{k,j}(\mathbb{U}[\ell])}{m_k[\ell] c_{k,k}(\mathbb{U}[\ell])}\right) + \frac{\beta_k \sigma_k}{m_k[\ell] c_{k,k}(\mathbb{U}[\ell])} \right). \tag{35}$$
2) Update dual network power $\mathbf{n}[\ell+1]$ for all k:
$$n_k[\ell+1] = \frac{\beta_k/\tau[\ell]}{\Gamma_k^{\mathsf{DN}}(\mathbf{n}[\ell], \mathbb{U}[\ell])} n_k[\ell]. \tag{36}$$
3) Normalize $\mathbf{n}[\ell+1]$:
$$\mathbf{n}[\ell+1] \leftarrow \frac{\bar{P}}{\boldsymbol{\sigma}^{\mathsf{T}}\mathbf{n}[\ell+1]}\mathbf{n}[\ell+1]. \tag{37}$$
4) Update transmit beamformer $\mathbb{U}[\ell+1]$ for all k:
$$\mathbf{u}_k[\ell+1] = \mathcal{P}\left\{ \left(\sum_{j\neq k} n_j[\ell+1]\Sigma_{j,k} + w_k \mathbf{I} \right)^{-1} \Sigma_{k,k} \right\}, \tag{38}$$
where $\mathcal{P}\{\cdot\}$ is the operator that computes the dominant eigenvector of a matrix.
5) Update primal network power $\mathbf{m}[\ell+1]$ for all k:
$$m_k[\ell+1] = \frac{\beta_k/\tau[\ell]}{\Gamma_k^{\mathsf{PN}}(\mathbf{m}[\ell], \mathbb{U}[\ell+1])} m_k[\ell]. \tag{39}$$
6) Normalize $\mathbf{m}[\ell+1]$:
$$\mathbf{m}[\ell+1] \leftarrow \frac{\bar{P}}{\mathbf{w}^{\mathsf{T}}\mathbf{m}[\ell+1]}\mathbf{m}[\ell+1]. \tag{40}$$
7) Go to Step 1 until $\mathbf{m}[\ell], \mathbf{n}[\ell], \mathbb{U}[\ell], \tau[\ell]$ converge.
8) Initialize power in (9) $\mathbf{p}[0] = \mathbf{m}[\ell]$. Run Algorithm 1 using the converged solution $\mathbb{U}$ at Step 7 until $\mathbf{p}[\ell]$ converges.

C. Algorithm Design

From the aforementioned analysis, we present a decentralized algorithm, Algorithm 2, to compute the near-optimal solution of (7), i.e., $(\mathbf{p}^{\star}(\mathbb{U}^{\text{cem}}), \mathbb{U}^{\text{cem}})$, as follows:

Theorem 2: First, starting from any initial point $\mathbf{n}[0]$, $\mathbf{m}[0]$, and $\mathbb{U}[0]$, the $\mathbf{n}[\ell]$, $\mathbf{m}[\ell]$, $\tau[\ell]$, and $\mathbb{U}[\ell]$ in the first six steps of Algorithm 2 converge geometrically fast to the optimal solution $\mathbf{n}^{\text{cem}}$, $\mathbf{m}^{\text{cem}}$, τ^{cem}, and $\mathbb{U}^{\text{cem}}$ of the certainty-equivalent margin problem (23). Second, the last step guarantees that $\mathbf{p}[\ell]$

converges fast to the optimal power $\mathbf{p}^\star(\mathbb{U}^{\text{cem}})$ of (9) under the fixed $\mathbb{U}^{\text{cem}}$.

Proof: First, for a fixed τ, we define the mapping $f^{(2)}: \mathbb{R}_+^{K\times 1} \to \mathbb{R}_+^{K\times 1}$ as:

$$f_k^{(2)}(\mathbf{n}) = \min_{\mathbf{u}_k}\left(\frac{\beta_k \mathbf{u}_k^\dagger(\sum_{j\neq k} n_j \Sigma_{j,k} + w_k \mathbf{I})\mathbf{u}_k}{\tau \mathbf{u}_k^\dagger \Sigma_{k,k}\mathbf{u}_k}\right), \quad \forall k. \quad (41)$$

For each k, $f_k^{(2)}(\mathbf{n})$ is the pointwise minimum over an infinite set of affine functions of $\mathbf{n}$ (indexed by $\mathbf{u}_k$), so $f_k^{(2)}(\mathbf{n})$ is a concave function. Thus, $f^{(2)}(\mathbf{n})$ is a concave self-mapping of $\mathbf{n}$. Then, we define $\|\cdot\|_{\mathsf{DN}}$ on $\mathbb{R}_+^{K\times 1}$ as $\|\mathbf{n}\|_{\mathsf{DN}} = \sum_k \sigma_k |n_k|/\bar{P}$, which can be easily proven as a monotone norm. By the network duality, the uplink and downlink max-min weighted certainty-equivalent SINR problems have the same optimal value $\zeta^\star$. Hence, the normalized fixed-point iteration $\hat{f}_k^{(2)}(\mathbf{n}[\ell+1]) = (1/\|f_k^{(2)}(\mathbf{n}[\ell])\|_{\mathsf{DN}}) f_k^{(2)}(\mathbf{n}[\ell])$ converges geometrically fast to the solution of the following conditional eigenvalue problem:

$$\frac{\mathbf{n}^{\text{cem}}}{\zeta^\star} = \frac{\text{diag}(\boldsymbol{\beta}\circ\mathbf{g})\left(\mathbf{C}^\top\mathbf{n} + \mathbf{w}\right)}{\tau} = \left(f^{(2)}(\mathbf{n}^{\text{cem}})\frac{\boldsymbol{\sigma}^\top}{\bar{P}}\right)\mathbf{n}^{\text{cem}}, \quad (42)$$

according to [17, Th. 1] and [23]. The convergence property of the primal network power $\mathbf{m}$ can also be proven by following the same line of argument.

We obtain Steps 2−4 to calculate the virtual uplink power $\mathbf{n}$ and the beamformer $\mathbb{U}$. Since the optimal uplink beamformer $\mathbb{U}^{\text{cem}}$ given in (33) is also the optimal beamformer in downlink by the network duality, Steps 5 − 6 keep the beamformer fixed and update the downlink power $\mathbf{m}$ accordingly. Step 1 updates the outage-mapped threshold τ based on $\mathbf{m}$ and $\mathbb{U}$. Then, the alternate optimization given by the first six steps of Algorithm 2 converges geometrically fast to the optimal solution of (23) as $\lim_{\ell\to\infty}\mathbf{n}(\ell) = \mathbf{n}^{\text{cem}}$, $\lim_{\ell\to\infty}\mathbb{U}(\ell) = \mathbb{U}^{\text{cem}}$, $\lim_{\ell\to\infty}\mathbf{m}(\ell) = \mathbf{m}^{\text{cem}}$, and $\lim_{\ell\to\infty}\tau(\ell) = \tau^{\text{cem}}$.

Second, based on Theorem 1, $\mathbf{p}[\ell]$ can converge geometrically fast to the optimal solution of (9) for a given $\mathbb{U}^{\text{cem}}$. It means that Algorithm 2 can guarantee a geometrically fast convergence rate to a feasible solution of (7), $(\mathbf{p}^\star(\mathbb{U}^{\text{cem}}), \mathbb{U}^{\text{cem}})$. ■

Remark 3: The iterations in Algorithm 2 can be made distributed by message passing [24]–[26]. The normalization of the primal power (40) and the dual power (37) can be made distributed using gossip algorithms to compute $\mathbf{w}^\top\mathbf{m}[\ell+1]$ and $\boldsymbol{\sigma}^\top\mathbf{n}[\ell+1]$ at each user similar to Algorithm 1. The update of the outage-mapped threshold (35) and primal power (39) can be computed by keeping separate copies of the received powers $m_j[\ell+1]c_{k,j}(\mathbb{U}[\ell+1])$ for each user during downlink transmission. The update of the transmit beamformer (38) and dual power (36) can be computed by keeping separate copies of the received virtual powers $n_j[\ell+1]$ and the updated $\mathbb{U}[\ell+1]$ for each user during uplink transmission.

Remark 4: Note that both the computation for the update of power in the dual network and the primal network have the same complexity as Algorithm 1 in each iteration.

V. Analytical Justification of Network Duality for CEM

In this section, we provide an analytical justification of the network duality in the CEM problem using the geometric programming duality in [31]. Suppose we know the optimal value $\alpha^\star$ in (4), let us consider the following weighted total power minimization problem:

$$\begin{aligned} &\text{minimize} \quad \sum_{k=1}^{K}\frac{\beta_k w_k}{c_{k,k}\tau^\star}p_k \\ &\text{subject to } \mathbb{P}(\mathsf{SINR}_k(\mathbf{p},\mathbb{U}) < \beta_k) \le \alpha^\star, \quad k = 1,\ldots,K, \\ &\qquad\qquad p_k \ge 0, \quad \|\mathbf{u}_k\|^2 = 1\ \forall k, \\ &\text{variables}: \mathbf{p}, \mathbb{U}, \end{aligned} \quad (43)$$

where $\tau^\star = \log\dfrac{1}{1-\alpha^\star}$. Using the result of Lemma 1, (43) is transformed to a deterministic optimization problem as follows:

$$\begin{aligned} &\text{minimize} \quad \frac{1}{\tau^\star}\mathbf{1}^\top\text{diag}(\boldsymbol{\beta})\text{diag}(\mathbf{g})\text{diag}(\mathbf{w})\mathbf{p} \\ &\text{subject to} \quad \frac{\beta_k\sigma_k}{p_k c_{k,k}} + \sum_{j\neq k}\log\left(1 + \frac{\beta_k p_j c_{k,j}}{p_k c_{k,k}}\right) \le \tau^\star \quad \forall k, \\ &\qquad\qquad p_k > 0, \quad \|\mathbf{u}_k\|^2 = 1\ \forall k, \\ &\text{variables}: \mathbf{p}, \mathbb{U}. \end{aligned} \quad (44)$$

Next, since $\log(1+x) \le x$ for all nonnegative x, we approximate the outage constraints of (44) to get the following approximation problem:

$$\begin{aligned} &\text{minimize} \quad \frac{1}{\tau^\star}\mathbf{1}^\top\text{diag}(\boldsymbol{\beta})\text{diag}(\mathbf{g})\text{diag}(\mathbf{w})\mathbf{p} \\ &\text{subject to} \quad \frac{\beta_k\sigma_k}{c_{k,k}}\frac{1}{p_k} + \sum_{j\neq k}\frac{\beta_k c_{k,j}}{c_{k,k}}\frac{p_j}{p_k} \le \tau^\star \quad \forall k, \\ &\qquad\qquad p_k > 0, \quad \|\mathbf{u}_k\|^2 = 1\ \forall k, \\ &\text{variables}: \mathbf{p}, \mathbb{U}. \end{aligned} \quad (45)$$

Given the optimal beamformer $\mathbb{U}^\star$, notice that this approximation problem (45) can be viewed as a certainty-equivalent problem with SINR-like constraints having a set of specially-constructed outage-mapped thresholds that incorporate the CDI:

$$\begin{aligned} &\text{minimize} \quad \frac{1}{\tau^\star}\mathbf{1}^\top\text{diag}(\boldsymbol{\beta})\text{diag}(\mathbf{g})\text{diag}(\mathbf{w})\mathbf{p} \\ &\text{subject to} \quad \frac{c_{k,k}p_k}{\sum_{j\neq k}c_{k,j}p_j + \sigma_k} \ge \frac{\beta_k}{\tau^\star} \quad \forall k, \\ &\qquad\qquad \mathbf{p} > 0, \\ &\text{variables}: \mathbf{p}. \end{aligned} \quad (46)$$

At this point, we remark that solving (46) is equivalent to solving (23) in the sense that their optimal solution $\mathbf{p}$ are the same. Now, let $e^{\tilde{p}_k} = p_k$. Then, we get the following equivalent convex optimization problem:

$$\begin{aligned}&\text{minimize } \sum_{k=1}^{K} \frac{\beta_k w_k}{c_{k,k}\tau^\star} e^{\tilde{p}_k}\\&\text{subject to } \log\left(\frac{\beta_k\sigma_k}{c_{k,k}} + \sum_{j\neq k}\frac{\beta_k c_{k,j}}{c_{k,k}} e^{\tilde{p}_j}\right) - \tilde{p}_k \le \log\tau^\star,\\&\text{variables : } \tilde{\mathbf{p}}.\end{aligned} \tag{47}$$

In the following, we will demonstrate this uplink-downlink network duality of CEM through the Lagrange duality of (47) based on the approach in [31].

Theorem 3: The optimal power $\mathbf{p}^\star$ in (23) and the Lagrange multiplier $\boldsymbol{\nu}^\star$ in (47) satisfy:

$$\nu_k^\star = p_k^\star \left(\frac{\beta_k w_k}{c_{k,k}\tau^\star} + \sum_{j\neq k} \frac{\frac{\beta_j c_{j,k}}{c_{j,j}}\nu_j^\star}{\frac{\beta_j\sigma_j}{c_{j,j}} + \sum_{i\neq j}\frac{\beta_j c_{j,i}}{c_{j,j}} p_i^\star} \right), \quad \forall k. \tag{48}$$

Furthermore, the following iterative updates can be used to compute $\boldsymbol{\nu}^\star$:

$$\mathbf{p}(\ell+1) = \mathbf{F}\mathbf{p}(\ell) + \operatorname{diag}\left(\frac{\boldsymbol{\beta}\circ\mathbf{g}\circ\boldsymbol{\sigma}}{\tau^\star}\right), \tag{49}$$

$$\mathbf{q}(\ell+1) = \mathbf{F}^\top\mathbf{q}(\ell) + \operatorname{diag}\left(\frac{\boldsymbol{\beta}\circ\mathbf{g}\circ\mathbf{w}}{\tau^\star}\right), \tag{50}$$

and:

$$\nu_k(\ell+1) = q_k(\ell+1)p_k(\ell+1), \quad k = 1,\ldots,K, \tag{51}$$

where $\mathbf{F}$ is the matrix with entries:

$$F_{k,j} = \begin{cases} 0, & \text{if } k = j\\ \frac{\beta_k c_{k,j}}{c_{k,k}\tau^\star}, & \text{if } k \neq j. \end{cases} \tag{52}$$

Proof: Note that the optimal solution $\mathbf{p}^\star$ of the CEM (23) is the same as the optimal primal solution of (46). At optimality, we have:

$$\frac{c_{k,k}p_k^\star}{\sum_{j\neq k} c_{k,j}p_j^\star + \sigma_k} = \frac{\beta_k}{\tau^\star}, \quad k = 1,\ldots,K. \tag{53}$$

which is also the optimal condition of (23). Next, we introduce nonnegative Lagrange multipliers $\boldsymbol{\nu}$ and write the Lagrangian function of (47):

$$\begin{aligned} L(\tilde{\mathbf{p}},\boldsymbol{\nu}) &= \sum_{k=1}^{K}\frac{\beta_k w_k}{c_{k,k}\tau^\star}e^{\tilde{p}_k} - \sum_{k=1}^{K}\nu_k(\tilde{p}_k + \log\tau^\star)\\ &\quad + \sum_{k=1}^{K}\nu_k\log\left(\frac{\beta_k\sigma_k}{c_{k,k}} + \sum_{j\neq k}\frac{\beta_k c_{k,j}}{c_{k,k}}e^{\tilde{p}_j}\right). \end{aligned} \tag{54}$$

The stationarity of the Lagrangian in the optimality conditions leads to:

$$\begin{aligned}\frac{\partial L}{\partial \tilde{p}_k} &= \frac{\beta_k w_k}{c_{k,k}\tau^\star}e^{\tilde{p}_k^\star} - \nu_k^\star + \sum_{j\neq k}\frac{\frac{\beta_j c_{j,k}}{c_{j,j}}\nu_j^\star e^{\tilde{p}_j^\star}}{\frac{\beta_j\sigma_j}{c_{j,j}} + \sum_{i\neq j}\frac{\beta_j c_{j,i}}{c_{j,j}}e^{\tilde{p}_i^\star}}\\ &= 0. \end{aligned} \tag{55}$$

Finally, we can get (48) by changing the variables from $\tilde{\mathbf{p}}^\star$ back to $\mathbf{p}^\star$. ∎

Note that, in Theorem 3, $\nu_k^\star = q_k^\star p_k^\star$ for all k. While $\mathbf{p}$ is the primal variable in (23) and $\boldsymbol{\nu}$ is the dual variable of (47), note that $\mathbf{q}$ is to be regarded as the auxiliary variable assisting with the computation of the primal and dual variables to (47). However, it is interesting to note that $\mathbf{q}^\star$ is also the optimal solution of the problem:

$$\begin{aligned}&\text{maximize } \mathbf{v}^\top\mathbf{q}\\&\text{subject to } \mathbf{q} \ge \mathbf{F}^\top\mathbf{q} + \operatorname{diag}\left(\frac{\boldsymbol{\beta}\circ\mathbf{g}\circ\mathbf{w}}{\tau^\star}\right),\\&\text{variables : } \mathbf{q},\end{aligned} \tag{56}$$

which is the Lagrange dual problem of a linear program that can be obtained from (46). Incidentally, the solution of (56) corresponds to the optimal solution of the virtual dual uplink network in (32) because of the optimality condition:

$$q_k^\star = \sum_{j\neq k}\frac{\beta_j c_{j,k}}{c_{j,j}}q_j^\star + \frac{\beta_k w_k}{c_{k,k}\tau^\star}, \quad k = 1,\ldots,K. \tag{57}$$

Therefore, this establishes the uplink-downlink network duality between (36) and (39) in Algorithm 2. In other words, the iterates of Algorithm 2 satisfy:

$$\lim_{k\to\infty}\mathbf{m}(k)\circ\mathbf{n}(k) = \boldsymbol{\nu}^\star, \tag{58}$$

thus motivating the use of the uplink-downlink network duality of the CEM problem for the beamformer update in Step 4 of Algorithm 2.

VI. Numerical Evaluations

In this section, we conduct numerical studies to evaluate the performance of our proposed CDI-based algorithm, i.e., Algorithm 2, for solving the outage probability max-min fairness problem in (4). We employ the angular spread model [12], [32] to generate the covariance matrices for the multiuser MISO network. Transmit angular spreads varying from 5 to 20 degrees across the links are assumed and the number of scatters is assumed to be 100. The desired signal links are centered at broadside, while the interfering links are centered at incident angles [12]. We assume the number of transmit antennas $N = 4$ and the number of links $K = 4$. The same weight and SINR threshold are also assumed.

Example 1: We first consider the convergence property of Algorithm 2. The total power $\bar{P}$ is held constant at 1 Watt and the SINR threshold is set to be 0.5. Figures 4 and 5 illustrate the evolution of the primal network power and the outage probability, respectively, for different users to verify the geometrical convergence of Algorithm 2 in the viewpoint of the power control, which have been proved in Theorems 1 and 2. It can be observed that Algorithm 1 adjusts the Link 2 to achieve the optimal solution. Empirically, the algorithm converges within $5 - 10$ iterations for different system parameter settings, which indicates the practical applicability of the algorithm. Figure 6 shows the convergence of the outage probability from a set of random feasible initial points for forty users using Algorithm 2. It can be seen that the convergence time does not increase much when we increase the number of users from four to forty, which demonstrates the fast convergence behavior of Algorithms 1 and 2.

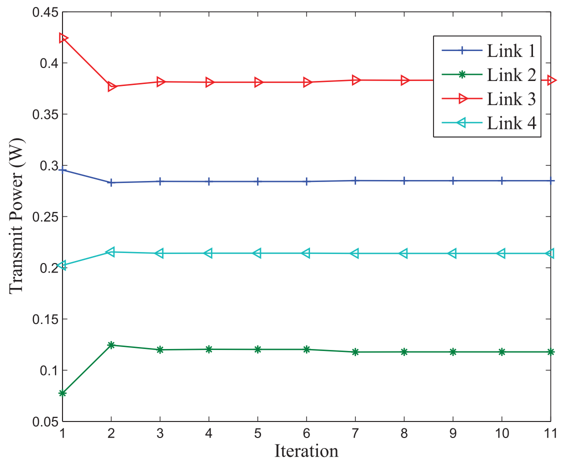

Fig. 4. Convergence result of the primal network transmit power for different users ($K = 4$, $N = 4$, $\bar{P} = 1$ W, SINR Threshold $= 0.5$).

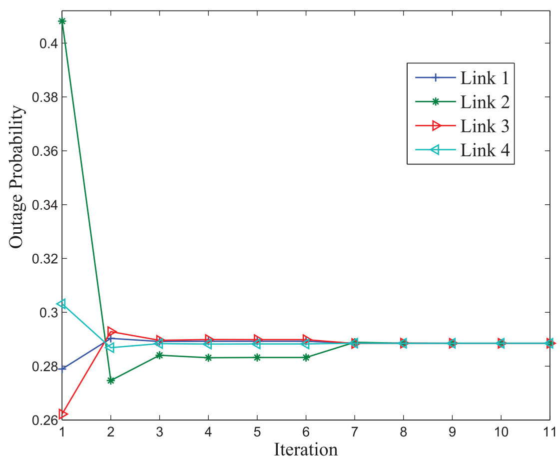

Fig. 5. Convergence result of the outage probability for different users ($K = 4$, $N = 4$, $\bar{P} = 1$ W, SINR Threshold $= 0.5$).

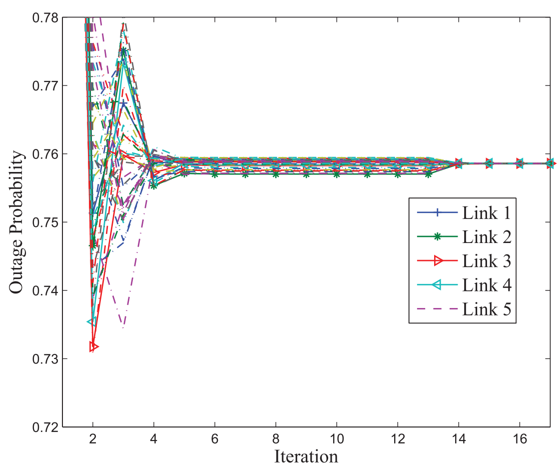

Fig. 6. Convergence of outage probability from random initial points for forty users using Algorithm 2.

We next demonstrate the effect of the total transmit power $\bar{P}$ and the SINR threshold on the maximum outage probability in the multiuser network. We average the maximum outage probability by considering 100 independent realizations of the

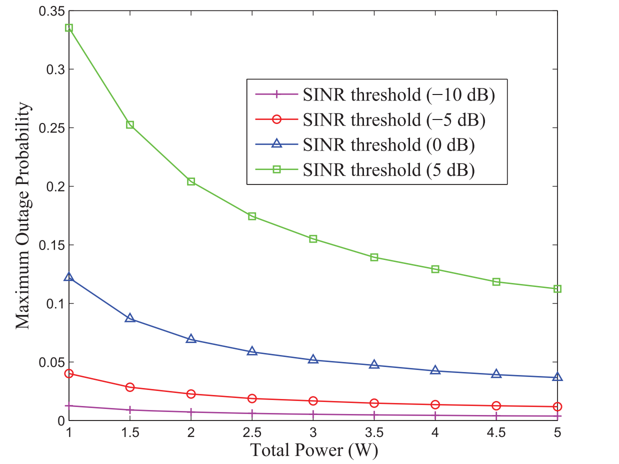

Fig. 7. The effect of total power and the SINR threshold on the maximum outage probability in the network ($K = 4$, $N = 4$, SINR Threshold $=$ $0.1, 0.5, 1, 2$).

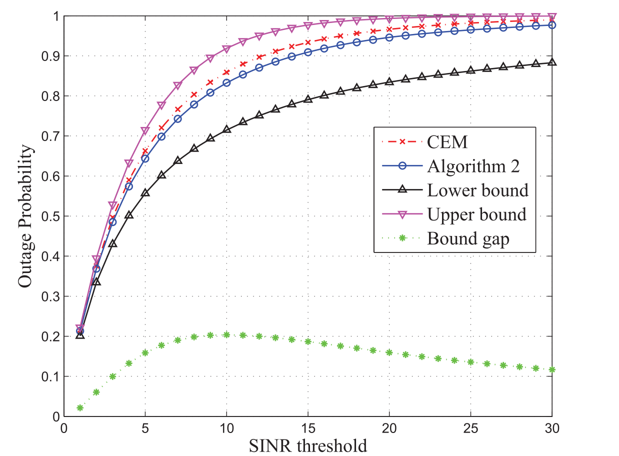

Fig. 8. The outage probabilities versus the SINR threshold ($K = 4$, $N = 4$, $\bar{P} = 12$ W). The performance of Algorithm 2 depends on the SINR threshold.

covariance matrices in the network. From Figure 7, we observe that by using the joint power control and beamformer design, a small maximum outage probability can be achieved for reasonably small SINR thresholds. When the network has a high predefined threshold, the total transmit power has to be increased considerably in order to reduce the outage probability. We also observe that the maximum outage probability monotonically decreases with the iteration of the algorithms.

Example 2: Second, we compare the solution obtained by Algorithm IV-C with the certainty-equivalent margin method and the lower and upper bounds. We consider the network in which the noise $\boldsymbol{\sigma}$ and the weight $\mathbf{w}$ are set randomly while the covariance matrices $\Sigma_{k,j}$ must be Hermitian and positive semi-definite. $\boldsymbol{\beta}$ and $\mathbf{w}$ are still set to be the same for all links.

We study how the SINR threshold affects the outage probabilities by letting $\boldsymbol{\beta}$ increase case by case from a weak signal case 1 (i.e., less than 3) to a stronger signal case 30 (i.e., larger than 25). A plot of the outage probabilities versus the SINR threshold is shown in Figure 8, which shows that Algorithm 2 can provide a solution close to the lower bound for the low outage probability. It illustrates that the curve

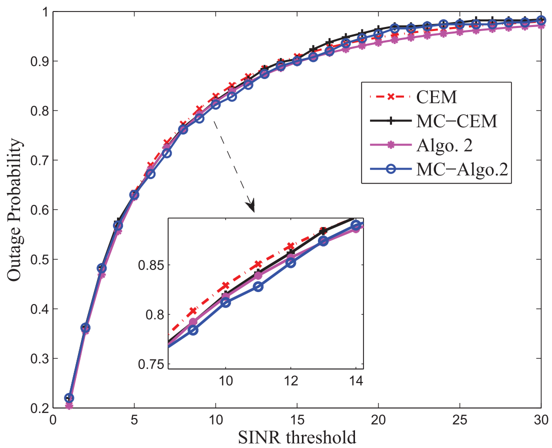

Fig. 9. The comparison between Algorithm 2 (Algo. 2) with the Monte-Carlo (MC) averaging simulations with parameters ($K = 4$, $N = 4$, $\bar{P} = 12$ W).

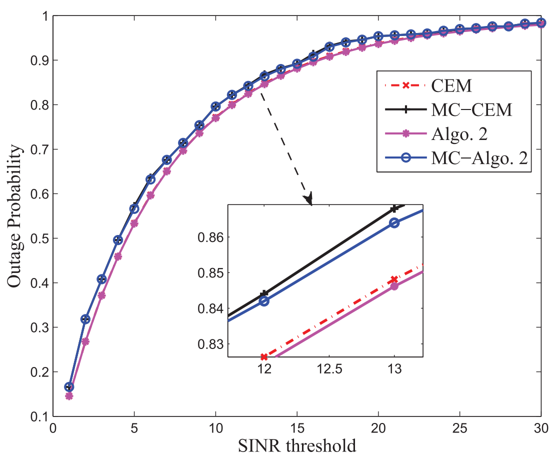

Fig. 10. The comparison between Algorithm 2 (Algo. 2) with the Monte-Carlo (MC) averaging simulations with parameters ($K = 10$, $N = 10$, $\bar{P} = 50$ W).

representing $O(\mathbf{p}^\star(\mathbb{U}^{\text{cem}}), \mathbb{U}^{\text{cem}})$ is basically identical to that of $O(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}})$. In addition, $(\mathbf{p}^\star(\mathbb{U}^{\text{cem}}), \mathbb{U}^{\text{cem}})$ is seen to be a better solution than $(\mathbf{p}^{\text{cem}}, \mathbb{U}^{\text{cem}})$. To validate the theoretical results, we compare the outage probability based on Monte-Carlo average simulations and averaging for every 300 Monte-Carlo runs. For each Monte-Carlo run, we randomly produce the channel vector of the fading, and an outage of the k-th user occurs if SINR_k is below β_k. Figures 9 and 10 show that our Algorithm 2 achieves better performance than the optimization of the certainty-equivalent margin problem on Monte-Carlo simulations. When the outage probability is small, the overall optimal value is close enough to the Monte-Carlo simulations.

VII. Conclusion

In this paper, we studied the use of statistical channel distribution information to optimize jointly the power and beamformer for transmission in a multiuser MISO network. This is useful to reduce feedback overhead in a network with many users. We studied the outage probability max-min fairness problem under a weighted sum power constraint. We first examined the special case under which the optimal set of beamformer was fixed, and presented a fast and decentralized algorithm using the nonlinear Perron-Frobenius theory. Then, to tackle the general case, we analyzed a certainty-equivalent margin counterpart, and proposed a near-optimal iterative algorithm that leveraged a network duality to decouple the nonconvex coupling between the beamforming and the power variables. We demonstrated that the low complexity requirement in channel distribution information-based transmission and the fast convergence performance of our algorithms make it ideal for practical implementation in very large multiuser networks.

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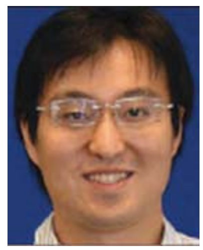

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