Fast Admission Control and Power Optimization with Adaptive Rates for Communication Fairness in Wireless Networks

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***Abstract*—Along with the exponentially increasing quantity of intelligent terminals connected to the Internet, the spectrum competition among users becomes more and more severe in wireless networks. The network have not the ability to satisfy all communication requirements due to the significantly increasing users and demanded rates. Energy-aware admission control has been proved to be an efficient way to tackle the infeasibility caused by the severe spectrum competition among users. However, the traditional admission control is limited by gradually removing chosen users, and pays less attention to the fairness. In this paper, we elaborate the concept of the fairness in a max-min optimization problem with respect to the transmission rates, by leveraging the model of bit error rates with Q-function for general fading communications. Then, we make use of the max-min rate fairness to smartly determine the subset of users to be admitted in wireless networks. Meanwhile, the overall energy consumption is minimized and the network fairness is guaranteed. In particular, the algorithms can tackle more than one user at each iteration. Numerical evaluations show the effectiveness of the algorithms.**

***Index Terms*—Max-min rate fairness, power optimization, admission control, bit error rate, Q-function.**

I. Introduction

We are stepping into the era in which everything is connected to the Internet, including not only computers, mobile phones, but also cars, drones etc. Wireless applications often compete with each other to meet the corresponding rate demands due to the scarce bandwidth. The fairness in competition has been noticed for power optimization, when all users want their own rates to be as high as possible [1], [2]. However, when there are plenty of users as well as the demanded rates are very high, not all users are able to transmit at their demanded rates simultaneously, due to the multiuser interferences in hostile radio environments [3]–[5]. Therefore, there exists an interesting issue: how to consume minimum energy but to support maximum intelligent terminals which can transmit at their demanded rates simultaneously. Energy-aware admission control has been regarded as a promising approach to guarantee the demands of admitted users [6]–[8], but pays little attention on the fairness among users. It is important to study how to fairly admit users with joint optimization. In addition, the decentralized control is needed for large-scale wireless networks, as the centralized approach may be costly and time consuming.

In this paper, the fairness is reflected by the optimal rate value of the end-to-end weighted max-min rate problem, in which the weights represent the flexible artificial factors and the data rate function is based on the bit error rates (BER) related with the Signal to Interference Noise Ratio (SINR) and the tabulated Q-function [9]–[11]. We design an iterative rate control algorithm, which converges geometrically fast, to obtain the optimal rate fairness by leveraging the nonlinear Perron-Frobenius theorem. Then, the optimal max-min rate fairness can be applied as the eligibility criteria for the admission to denser wireless networks. When the wireless networks can satisfy the rate demands of all users, the fairness is underground and energy consumption becomes increasingly important due to the power budgets of intelligent terminals [12]. We tackle the total power minimization problem by using a distributed algorithm to achieve the optimal power allocation whenever the problem is feasible.

When the wireless networks cannot fulfill the rate demands for all users, the primary algorithm may not converge or may be unstable in general. Admission control is frequently used to deal with this infeasibility issue in the past. The traditional admission approaches greedily reject the chosen users one by one until the rest of users could achieve their rate demands. Aggressive admission control unduly removes users, leading to the under-utilized network, albeit with a lower total energy consumption. We propose fast decentralized algorithm based on the standard interference function framework [13], to optimize the energy consumption and tackle the infeasibility by adapting the rate demands beyond the fairness. This leads to a decentralized dynamic approach without the need of a

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centralized admission controller.

In practice, it is unfair for the users with low rate demands. The channel quality of the user with a poor channel condition is critical as the allocated transmission power is constrained by the other users [14]. Intuitively, those users with high rate demands would be better to choose other more efficient communication networks. Moreover, the value of fairness is also varying with the admission control of the users in wireless networks, e.g., the value of fairness will increase when some unsatisfied users quit. Instead of admitting or rejecting one chosen user at each iteration, we deal with more users at each iteration by making use of the lower and upper bounds of the network fairness to classify the users into three categories.

Overall, the contributions of the paper are as follows:

1) Firstly, we analytically solve the minimal data rate maximization problem with power budgets, and propose a distributed algorithm to obtain the optimal max-min rate fairness.
2) Secondly, based on the established feasibility conditions, we propose a dynamic algorithm for the graceful handling of infeasibility in wireless networks. In particular, the algorithm optimizes the overall power consumption by adapting the rate demands to guarantee the fairness.
3) Thirdly, we propose a fast iterative admission control algorithm to admit users by exploiting the fairness algorithm as a sub-module.

The rest of the paper is organized as follows: We discuss the relevant state-of-the-art literatures in Section II. Then, we firstly formulate the max-min rate fairness problem for the wireless networks with the data rate functions related with SINR and Q-function in Section III. Secondly, we study the characteristics of this optimization problem at optimality, and propose a distributed power control algorithm to obtain the optimal solution in Section IV. Moreover, in Section V, we study the feasibility of a total power minimization problem, and design a dynamic power control algorithm that adapts the rate requirements to minimize the total energy consumption. In Section VI, we propose the fast admission control strategy by using the lower and upper bounds of the fairness. Then, we numerically evaluate the performance of our algorithms in Section VII. Finally, we conclude the work in Section VIII. All the proofs of Lemmas and Theorems can be found in the Appendix.

We adopt the following notations in this paper: Lowercase and uppercase boldfaces are used for vectors and matrices, respectively. The super-script $(\cdot)^\top$ denotes the transpose. $\|\cdot\|_2$ denotes the ℓ_2 norm (Euclidean norm). $e^{\mathbf{x}}$ and $\log \mathbf{x}$ denote $(e^{x_1},\dots,e^{x_n})^\top$ and $(\log x_1,\dots,\log x_n)^\top$, respectively.

II. Related Works

In the literatures, there has been extensive works on admission control to deal with the feasibility issue. In [3], Mahdavi-Doost et al. developed a centralized gradual removal algorithm that removes users to increase the maximum achievable SINR in the system. In [4], Rasti et al. proposed a distributed temporarily removal algorithm, in which users stop transmission once their instantaneous power consumptions exceed a certain threshold. Mitliagkas et al. [5] removed users based on convex relaxation to obtain an approximate solution for the feasibility of the networks. Halldorsson et al. proposed algorithms based on a novel linear programming formulation for admission control problems with constant-factor performance guarantees in [6]. The authors made use of Lagrange dual parameter to gradually remove the user who produced the highest interference in [7]. Zhao et al. jointly designed the transmission beamformers and power control for densely underlaid small cell access points in wireless backhaul in [8]. Meanwhile, there is room for improvement of efficient admission control considering the fairness among users.

Fairness amongst intelligent terminals has been analyzed as an important issue from several dimensions, e.g., energy usage, achieving required quality of services, spectrum sharing, and so on [1]. Kelly et al. firstly analyzed the stability and fairness for rate control algorithms in [15]. Tassiulas et al. proposed a fair scheduling which assigned dynamic weights to the flows as max-min fairness allocation of bandwidth in wireless ad-hoc networks in [16]. Rangwala et al. in [17] achieved a fair and efficient rate allocation in wireless sensor networks. Eryilmaz et al. in [18] fairly allocated resources for competing users in the time-varying channel conditions. Zheng et al. in [19] proposed a distributed approach with tuning fair parameters to generalize the diverse set of link rate functions and constraints.

In contrast to the commonly used two timescale approach (finding a maximum user set first before minimizing the total energy consumption of all users in the set) in earlier works, we propose a single timescale decentralized power control algorithm with low complexity. We make use of the network fairness obtained from a max-min rate fairness optimization problem, to adapt the demanded rates of users when the system is infeasible. We develop a novel power allocation approach that can jointly capture the energy consumption and rate fairness. Our approach accommodates a variety of extensions, including CDMA and Shannon capacity. Additionally, the proposed approach is computationally fast and scalable in a decentralized manner without parameter configuration. We classify the users into three strategies and deal with more users at each iteration. This is useful for practical wireless networks with a large number of users.

III. System Model

Consider a wireless network with L source-destination pairs. In the physical layer, a common spectrum bandwidth is shared by all the users in the network. The transmission power of the transmitter of link l is denoted by p_l. The received SINR at the receiver of link l is given in terms of the transmission power $\mathbf{p}=(p_1,\dots,p_L)^\top$ as:

$$\mathsf{SINR}_l(\mathbf{p}) = \frac{G_{ll}p_l}{\sum\limits_{j\neq l} G_{lj}p_j + \sigma_l}, \tag{1}$$

where G_{lj} is the channel gain from the transmitter of link j to the receiver of link l, and σ_l is the additive white Gaussian noise (AWGN) at the receiver of link l. The channel gain matrix $\mathbf{G}$ takes propagation loss, spreading loss and other transmission modulation factors into consideration.

As the channel fading significantly affects the performance of wireless networks, we model the achievable data rate of link l by [20], [21]:

$$f_l(\mathsf{SINR}_l(\mathbf{p})) = R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right), \tag{2}$$

where R is the fixed data transmission rate, and the Q-function of BER is defined as:

$$Q(x) = \frac{1}{\sqrt{2\pi}}\int_x^{\infty} e^{-y^2/2}dy. \tag{3}$$

In particular, our proposed algorithms in this paper are also applied to the conventional rate functions, e.g., $\mathsf{SINR}_l(\mathbf{p})$ for CDMA approximation and $\log(1 + \mathsf{SINR}_l(\mathbf{p}))$ for Shannon capacity formula [19]. In general, it is very difficult to analyze most of modulation schemes as the properties of involved functions are nonconvex or would not translate into the achievable rates. For the other modulation and coding schemes, our proposed algorithms could still run but the convergence may not be guaranteed. As the required properties of the involved may not hold, it would produce unpredictable results, even has the possibility of divergence or instability.

In practice, there is typically a budget for the transmit power allowed to each transmitter. We assume that the feasible set of powers can be represented by a linear constraint:

$$\mathbf{A}\mathbf{p} \le \bar{\mathbf{p}}, \tag{4}$$

where $\bar{\mathbf{p}}$ is a $N \times 1$ positive vector of constraint values and $\mathbf{A}$ is a $N \times L$ nonnegative weight matrix. Note that every link involves at least one power constraint. Moreover, even if the transmit power of a link is physically unconstrained, we could simply augment $\mathbf{A}$ and $\bar{\mathbf{p}}$ with an arbitrarily large individual power constraint on that link. Two possible scenarios included under (4) are that of individual power constraints on each link in an uplink network (with $N = L$ and $\mathbf{A} = \mathbf{I}$), and that of a single total power constraint on all links in a downlink network (with $N = 1$ and $\mathbf{A} = \mathbf{1}^\top$). Linear power constraints also appear in other kinds of ad-hoc settings, e.g., interference temperature constraints in wireless cognitive radio settings.

Since the various users could have different service requirements, the network could assign different weights to the users for the rate allocation. Let β_l denote the weight assigned to the l-th user. The weighted max-min rate fairness problem is formulated as:

$$\begin{array}{ll} \text{maximize} & \min\limits_l \dfrac{f_l(\mathsf{SINR}_l(\mathbf{p}))}{\beta_l} \\ \text{subject to} & \mathbf{A}\mathbf{p} \le \bar{\mathbf{p}}, \\ & \mathbf{p} \ge \mathbf{0}, \\ \text{variables :} & \mathbf{p}. \end{array} \tag{5}$$

Then, we derive the optimal fairness solution of (5) first, and connect (5) to the total energy minimization problem for a given certain weight $\boldsymbol{\beta}$. Then, we address the feasibility issue by adapting the rate demands and leveraging the admission control. Figure 1 gives an overview of the developments and the related optimization problems in this paper.

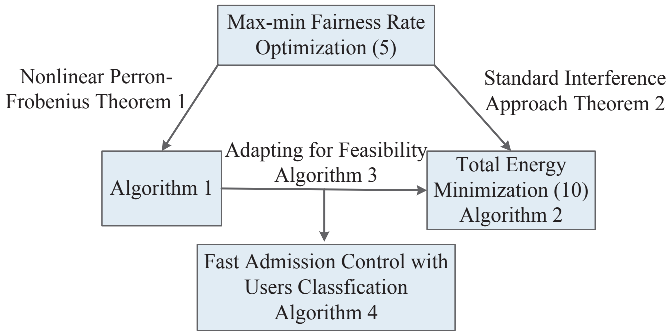

Fig. 1. An overview of optimization problems and iterative algorithms.

IV. Distributed Power Control Algorithms

By introducing an auxiliary variable τ, we rewrite (5) as:

$$\begin{array}{ll} \text{maximize} & \tau \\ \text{subject to} & \dfrac{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right)}{\beta_l} \ge \tau, \quad l = 1, \dots, L, \\ & \mathbf{A}\mathbf{p} \le \bar{\mathbf{p}}, \\ & \mathbf{p} \ge \mathbf{0}, \\ \text{variables :} & \tau, \mathbf{p}. \end{array} \tag{6}$$

Intuitively, we could promote the communication rate by increasing the transmit power until no further more. Therefore, we have the following result.

Lemma 1: The optimal power solution $\mathbf{p}^\star$ of (6) satisfies that there is at least one link l such that $(\mathbf{A}\mathbf{p}^\star)_l = \bar{p}_l$.

Proof: See Appendix A. ■

Then, max-min rate problem will achieve the fairness just as a result of “Bucket Theory”. Therefore, we have the following result.

Lemma 2: $\dfrac{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}^\star)}\right)\right)}{\beta_l}$ for all l are equal.

Proof: See Appendix B. ■

Now we propose the following algorithm. Let k index the iteration number.

Algorithm 1: Max-min Rate Fairness Control Algorithm.

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1) Initialize an arbitrary $\mathbf{p}(0)$ in feasible power budgets and a small positive ϵ.
2) Update the transmit power for each link:
$$p_l(k+1) = \frac{\beta_l}{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}(k))}\right)\right)} p_l(k). \tag{7}$$
3) Normalization of power:
$$p_l(k+1) = \left(\min_j \frac{\bar{p}_j}{(\mathbf{A}\mathbf{p}(k+1))_j}\right) p_l(k+1). \tag{8}$$
4) Go to Step 2 until $\|\mathbf{p}(k+1) - \mathbf{p}(k)\|_2 \le \epsilon$.

Theorem 1: $\mathbf{p}(k)$ in Algorithm 1 converges to an optimal solution of (6) from any $\mathbf{p}(0) > \mathbf{0}$. Moreover, $\mathbf{p}(k)$ converges geometrically fast.

Proof: See Appendix C. ∎

Remark 1: The power update (7) is distributed based on the current power and rate for l-th user, as $R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}(k))}\right)\right)$ denotes the current rate. The normalization (8) can be made distributed using gossip algorithms [22]–[24]. The necessity of transmitting information at each iteration may slow down the convergence.

Remark 2: The function (7) is concave when the value of SINR is less than two, that theoretically guarantees the convergence of Algorithm 1. It is interesting to note that the following second-order derivative is below a small positive threshold ϵ and:

$$\begin{aligned} &\lim_{\mathsf{SINR}_l(\mathbf{p})\to\infty} \nabla^2 \frac{\beta_l}{R(1-2Q(\sqrt{\mathsf{SINR}_l(\mathbf{p})}))}\mathsf{SINR}_l(\mathbf{p}) \\ =\ &\lim_{\mathsf{SINR}_l(\mathbf{p})\to\infty} \frac{\sqrt{\mathsf{SINR}_l(\mathbf{p})}}{e^{\mathsf{SINR}_l(\mathbf{p})}} \\ =\ &0, \end{aligned} \tag{9}$$

which leads to the convergence of Algorithm 1 for the large value of SINR.

V. Total Energy Consumption

In this section, we consider the problem of minimizing the total energy consumption subject to given data rate requirements and individual power budgets in the wireless networks, which is formulated as follows:

$$\begin{aligned} \text{minimize}\quad & \sum_{l=1}^{L} p_l \\ \text{subject to}\quad & R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right) \geq \bar{r}_l, \quad l=1,\ldots,L \\ & p_l \leq \bar{p}_l, \quad l=1,\ldots,L \\ & \mathbf{p} \geq \mathbf{0}, \\ \text{variables}:\quad & \mathbf{p}, \end{aligned} \tag{10}$$

where $\bar{p}_l$ is the budget of the transmit power for the l-th user and $\bar{r}_l > 0$ is the rate demand of the l-th user. In general, (10) may or may not be feasible. This means that it may not be possible to have the data transmission rates of all the users to be larger than $\bar{\mathbf{r}}$ in (10).

We first address (10) by leveraging the standard interference function framework introduced in [13], [25] when (10) is feasible. We propose a distributed fixed-point algorithm to compute the optimal solution of (10).

Algorithm 2: Total Energy Minimization Algorithm.

$$p_l(k+1) = \min\left\{ \frac{\bar{r}_l}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}(k))}\right)\right)} p_l(k), \bar{p}_l \right\}. \tag{11}$$

Theorem 2: Starting from any initial point $\mathbf{p}(0)$, Algorithm 2 converges geometrically fast to an optimal solution of (10) when (10) is feasible.

Proof: See Appendix D. ∎

However, when (10) is infeasible, the power control has to tackle the infeasibility issue. It is interesting to note that the infeasibility is caused by high demanded rates $\bar{\mathbf{r}}$, due to the nature budget $\bar{\mathbf{p}}$ of the communication power. By leveraging the results in the previous sections, the users have the common rate fairness, i.e., the optimal value of (5) regardless of their own demanded rates $\bar{\mathbf{r}}$. In order to distinguish from the power $\mathbf{p}$ in (10), we introduce $\mathbf{z}$ as the auxiliary variable which denotes the virtual power corresponding to (5). In other words, (10) must be feasible if we adapt the demanded rates to be the optimal value of (5) for all users, i.e., $R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{z}^\star)}\right)\right)$ where $\mathbf{z}^\star$ denotes the optimal power solution of (5) obtained from Algorithm 1. Intuitively, the power can be further reduced if we keep the rate demands for the users whose demanded rates are less than the common rate fairness. Therefore, we propose the following adaptive algorithm to support all users.

Algorithm 3: Adaptive Energy Minimization Algorithm.

1) Initialize an arbitrary $\mathbf{p}(0)$, $\mathbf{z}(0)$ in feasible power budgets and a small positive ϵ.
2) Update the virtual power $\mathbf{z}(k+1)$:
$$z_l(k+1) = \frac{z_l(k)}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{z}(k))}\right)\right)}, \quad \forall l. \tag{12}$$
3) Normalization $\mathbf{z}(k+1)$:
$$z_l(k+1) = \left(\min_j \frac{\bar{p}_j}{z_j(k+1)}\right) z_l(k+1). \tag{13}$$
4) Update the transmit power of each user:
$$\begin{aligned} p_l(k+1) = \min\ &\left\{ \frac{\min\left\{\bar{r}_l, R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{z}(k))}\right)\right)\right\}}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}(k))}\right)\right)} \right. \\ &\left. \times p_l(k), \bar{p}_l \right\}. \end{aligned} \tag{14}$$
5) Go to Step 2 until $\|\mathbf{p}(k+1)-\mathbf{p}(k)\|_2 \leq \epsilon$.

Corollary 1: Starting from any initial point $\mathbf{z}(0)$ and $\mathbf{p}(0)$, $\mathbf{p}(k)$ in Algorithm 3 converges geometrically fast to the optimal solution of (10), by replacing the righthand side of the rate constraints in (10) with $\min\{\bar{r}_l, f_l(\mathsf{SINR}_l(\mathbf{z}^\star)\}$ where $\mathbf{z}^\star$ is the optimal solution of (5) toward the network fairness, i.e., the convergence of $\mathbf{z}(k)$.

Proof: See Appendix E. ∎

VI. Fast Admission Control with Users Classification

In this section, we focus on the admission control based on the optimal solution of (5) for different user sets. Finding the optimal set of users whose rate demands can be all satisfied is a NP-hard combinatorial problem [4]. When the number of users is large, it is not practical to examine all the combinations of the users to choose a feasible set with the minimum energy consumption. The state-of-art techniques for admission control [3], [4] use the heuristic greedy strategy to reject one user at each time. In the following, we propose a faster admission control procedure to admit and reject more users at each time while the rest part of algorithms follows the geometrical convergence.

Theorem 3: The optimal value of (5), i.e., the network fairness, increases when any of users in the networks are rejected.

Proof: See Appendix F. ■

From Algorithm 3, we have made use of the fairness among all the users in the networks. Intuitively, the users whose rate demands are below this fairness should be admitted into the communication networks. Actually, we can further deduce the upper bound of the fairness by Algorithm 1 for all the admitted users. It is interesting to note that the users are classified by the lower and upper bounds of the network fairness into three categories: 1) *admitted users* whose rate demands are below the lower bound of the network fairness regarding to both the admitted users and the adaptive users, 2) *adaptive users* whose rate demands may need to be adapted to the lower bound of the network fairness, and 3) *rejected users* whose rate demands are beyond the upper bound of the network fairness regarding to only the admitted users. Therefore, the following fast admission control algorithm admits or rejects more users at each time.

Algorithm 4: Fast Admission Control Algorithm.

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1) **Initialization:**
   - Initialize an arbitrary $\mathbf{p}(0)$ and $\mathbf{z}(0)$ in feasible power budgets, a small positive ϵ and the set of supported users $\mathcal{A}(0) = \emptyset$.
2) **Compute the lower bound of the network fairness considering all users (except the rejected users):**
   - Update the virtual power $\mathbf{z}(k+1)$:
$$z_l(k+1) = \frac{z_l(k)}{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{z}(k))}\right)\right)}, \quad \forall l. \tag{15}$$
   - Normalization $\mathbf{z}(k+1)$:
$$z_l(k+1) = \left(\min_j \frac{\bar{p}_j}{z_j(k+1)}\right) z_l(k+1). \tag{16}$$
   - Go to (15) until $\|\mathbf{z}(k+1) - \mathbf{z}(k)\|_2 \leq \epsilon$.
   - Admit the users into $\mathcal{A}(t)$:
$$\arg_l \bar{r}_l < \min_j R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_j(\mathbf{z}(k))}\right)\right), \tag{17}$$
     for all l, whose rate requirements are below the lower bound of the network fairness, i.e., the right part in (17).
3) **Compute the upper bound of the network fairness considering only admitted users:**
   - Set the virtual power $z_l(k) = 0$, $\forall l \notin \mathcal{A}(t)$:
   - Update the virtual power $\mathbf{z}(k+1)$:
$$z_l(k+1) = \frac{z_l(k)}{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{z}(k))}\right)\right)}, \quad \forall l. \tag{18}$$
   - Normalization $\mathbf{z}(k+1)$:
$$z_l(k+1) = \left(\min_j \frac{\bar{p}_j}{z_j(k+1)}\right) z_l(k+1). \tag{19}$$
   - Go to (18) until $\|\mathbf{z}(k+1) - \mathbf{z}(k)\|_2 \leq \epsilon$.
   - Reject the users for all l:
$$\arg_l \bar{r}_l > \max_j R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_j(\mathbf{z}(k))}\right)\right), \tag{20}$$
     whose rate requirements are beyond the upper bound of the network fairness, i.e., the right part in (20).
4) **Find the Feasible Set:**
   - Go to Step 2 to update the new lower bound (see Figure 2) after resetting the auxiliary variable z_l for the rest of users, i.e., the adaptive users, until there are no any more rejected users.
5) **Adaptive Energy Minimization:**
   - Update the transmit power of each admitted user and adaptive user:
$$p_l(k+1) = \min\left\{\frac{\bar{r}_l}{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}(k))}\right)\right)} p_l(k), \bar{p}_l\right\}. \tag{21}$$
   - Repeat until $\|\mathbf{p}(k+1) - \mathbf{p}(k)\|_2 \leq \epsilon$.
   - Update the transmit power of the adaptive users whose rate requirements are not satisfied:
$$p_l(k+1) = \min\left\{\frac{\min\left\{\bar{r}_l, R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{z}(k))}\right)\right)\right\}}{R\left(1 - 2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}(k))}\right)\right)} \times p_l(k), \bar{p}_l\right\}. \tag{22}$$
   - Repeat until $\|\mathbf{p}(k+1) - \mathbf{p}(k)\|_2 \leq \epsilon$.

Corollary 2: Starting from any initial point $\mathbf{z}(0)$ and $\mathbf{p}(0)$, $\mathbf{p}(k)$ in Algorithm 4 converges geometrically fast to the optimal solution of (10) with the admitted users and the adaptive users.

Proof: See Appendix G. ■

Remark 3: The power updates (21), (22) are distributed based on the current power and rate for the l-th user. The virtual power updates (15), (18), normalization (16), (19) and classifications (17), (20) can be made distributed using gossip algorithms [22]–[24].

Remark 4: When the total power minimization problem in (10) is infeasible, the output from Algorithm 4 satisfies the power budgets, and whether it satisfies the rate constraints set in (10) depends on the result of feasible set obtained by Step 4 in Algorithm 4. When all the adaptive users' rate requirements are satisfied, the output from Algorithm 4 satisfies both the power budgets and the rate constraints. In contrary, if there are any adaptive users which need to adjust their rate requirements to the lower bound of the network fairness, the output from Algorithm 4 only satisfies the power budgets. The example is shown in the next experimental section.

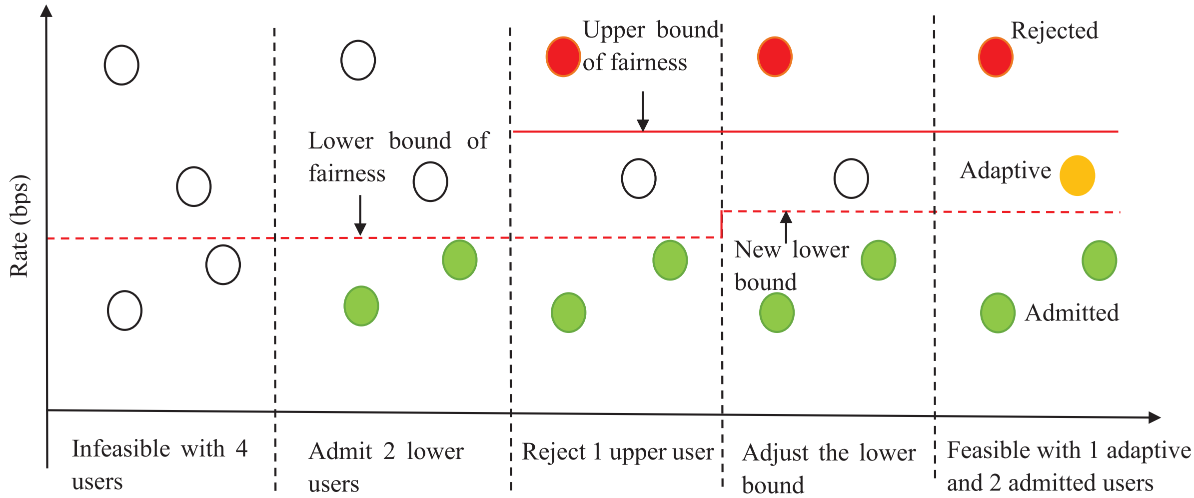


Fig. 2. An illustration of Algorithm 4 for fast admission control when (10) is infeasible. The green and red circles denote the admitted and rejected users, respectively. The yellow circle denotes the adaptive user whose rate requirement may need to be adapted.

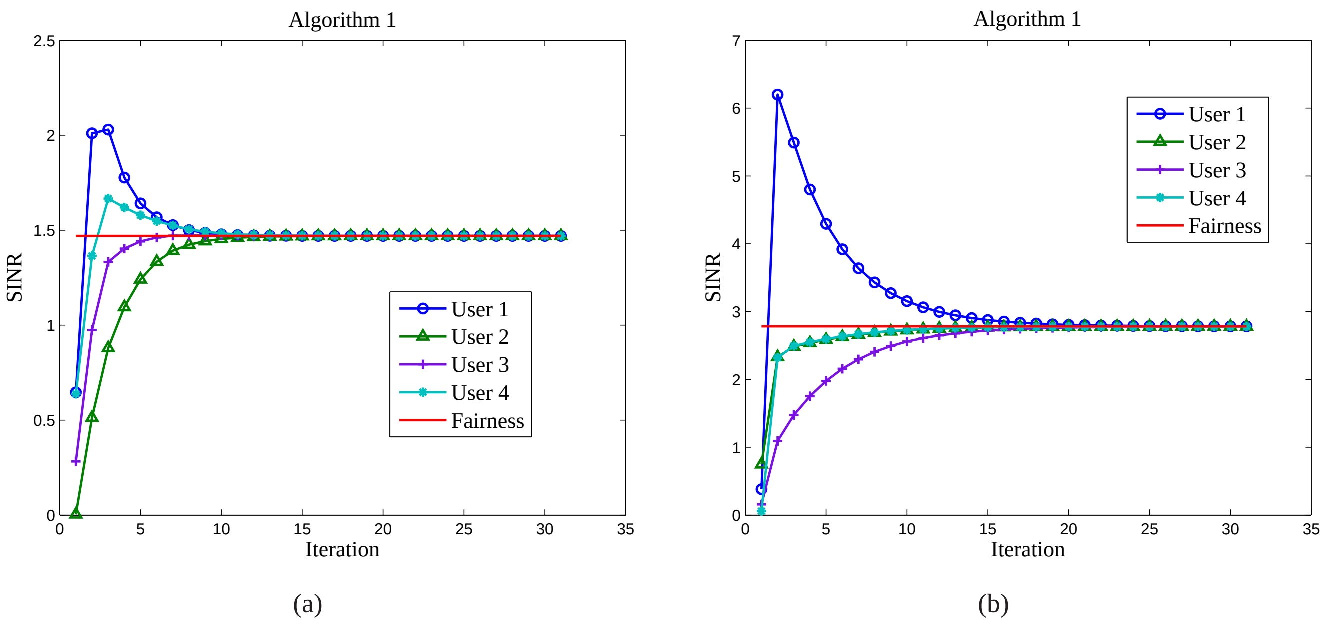


Fig. 3. Illustrations of the convergence of Algorithm 1 where the red line (i.e., Fairness) is the obtained optimal value of (5). The convergence is shown for the small and large values of SINR in (a) and (b), respectively.

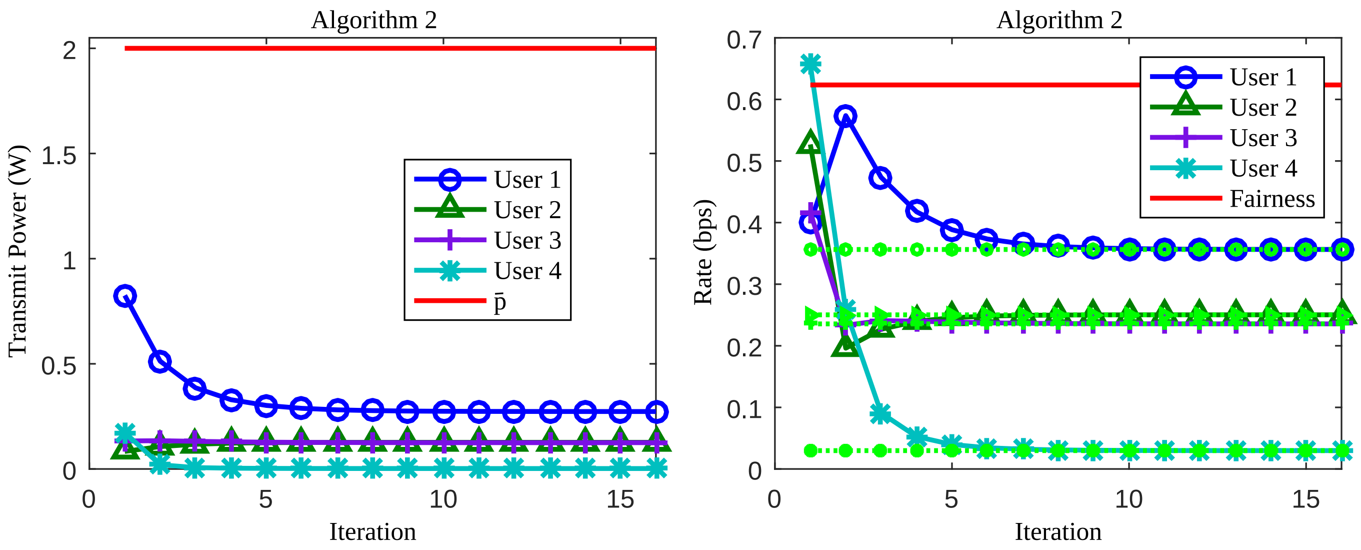
VII. Numerical Examples

In this section, we evaluate the performance of the proposed algorithms numerically. The fixed c ommunication rate is set as $R = 1$ Mbps. The AWGN at the receiver, i.e. σ_l, is assumed to be 5×10^{-3} W. The channel gain is adopted from the well-known model $G_{jk} = kd_{jk}^{-4}$ [26], where d_j is the distance between the j-th transmitter and its receiver, and $k = 0.09$ is the attenuation factor that represents power variations due to path loss. The budgets of the transmit power for all users are the same, i.e., $\bar{p}_l = 2$ W for all l.

Example 1: We first show the convergence of Algorithm 1 through four users in a single-cell environment. Without loss of generality, we set the weight of all the users as the same, i.e., $\beta_1 = \cdots = \beta_L = 1$. Moreover, we use $\mathbf{A} = \mathbf{I}$ to consider the individual power constraints for the convenience of the comparison.

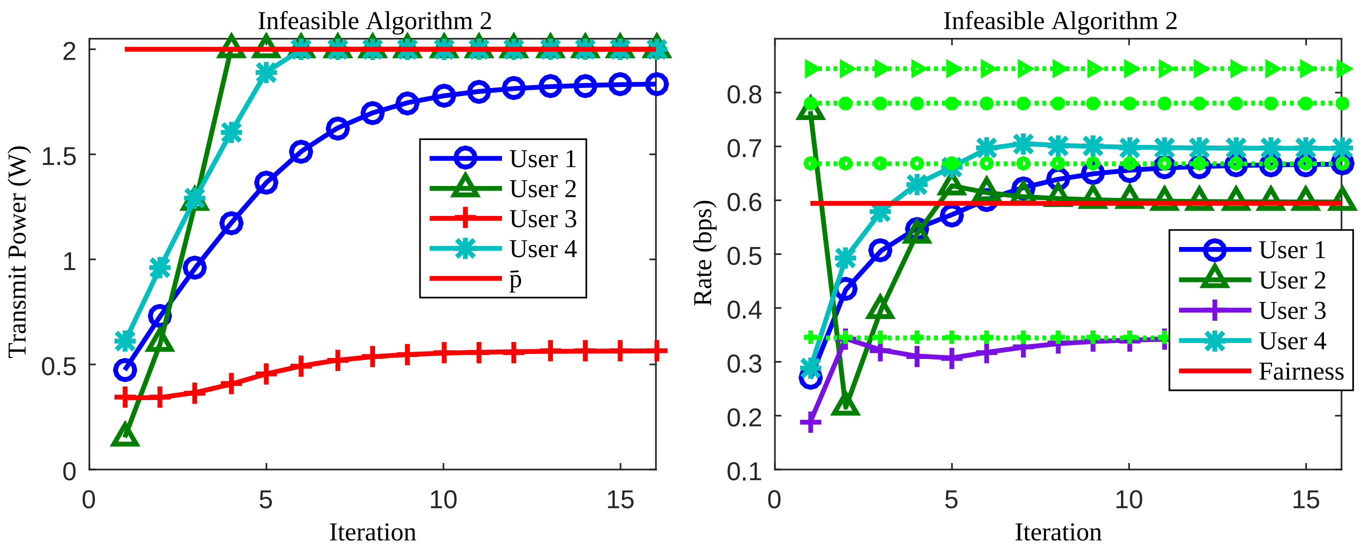
To evaluate the convergence of Algorithms 1 for the large SINR, we demonstrate the evolution of Algorithms 1 in terms of SINR instead of the rate. Figure 3 (a) verifies the geometrical convergence of Algorithm 1 for the small value of SINR. It can be seen that Algorithm 1 can still converge to the optimal value named as the fairness for the large value of SINR in Figure 3 (b). Compared to the small SINR which is below than two, the speed of convergence is a little slower for the large SINR. Note that we can get the corresponding individual rate based on given SINR from (2), and the network is always feasible for the max-min rate fairness problem (5).

Example 2: Secondly, we show the convergence of



(a) (b)

Fig. 4. Illustrations of the convergence of Algorithm 2 for feasible (10) in terms of transmit power in (a) and individual rate in (b), respectively. Each green dashed line represents the corresponding demanded transmission rate for each user in (b).



(a) (b)

Fig. 5. Illustrations of the convergence of Algorithm 2 for infeasible (10) in terms of transmit power in (a) and individual rate in (b), respectively. Each green dashed line represents the corresponding demanded transmission rate for each user in (b).

Algorithm 2 through four users in the same environment as Example 1. The required rate vector is $\bar{\mathbf{r}} = [0.356, 0.25, 0.236, 0.03]$, which leads to the feasible (10). Figure 4 verifies that Algorithm 2 geometrically converges to the optimal solution of (10) when the network is feasible.

Note that the total energy minimization problem (10) may be infeasible, e.g., the required rate vector is $\bar{\mathbf{r}} = [0.668, 0.844, 0.345, 0.78]$. Algorithm 2 turns into a greedy algorithm that converges to a point, where Users 2 and 4 transmit at their maximum power level but still do not achieve their rate requirements as shown in Figure 5. Algorithm 2 also converges when (10) is infeasible, but to an unpredictable solution where not all of the users can satisfy their rate requirements. It is observed that the users (e.g., User 3 whose rate requirement is below the fairness obtained by Algorithm 1) can achieve its rate requirement. In contrary, the users (e.g., Users 2 and 4 whose rate requirements are beyond the fairness obtained by Algorithm 1) cannot achieve their rate requirements. However, User 1 achieves its rate requirement because User 3 transmits only on its rate demands instead of the fairness of the network.

Example 3: It is possible to obtain different feasible sets from different algorithms, and it is interesting to obtain the maximum feasible set which may also not be unique. Let the required rate vector be $\bar{\mathbf{r}} = [0.668, 0.186, 0.736, 1.28]$ which leads to the infeasible (10). Then, we can support all the users by adapting some users' rate demands to be $\bar{\mathbf{r}}^* = [0.628, 0.186, 0.628, 0.628]$. Figure 6 shows that Algorithm 3 converges to the solution of (10), where User 2 reaches its corresponding rate requirement which is below the fairness,

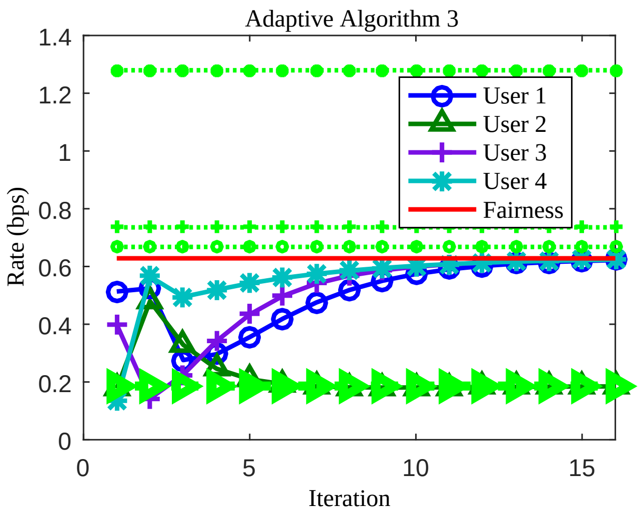


Fig. 6. An illustration of the convergence of Algorithm 3 for adaptive power allocation in terms of individual rate when (10) is infeasible. The red line (i.e., Fairness) is the optimal value obtained by Algorithm 1. Each green dashed line represents the corresponding demanded transmission rate for each user.

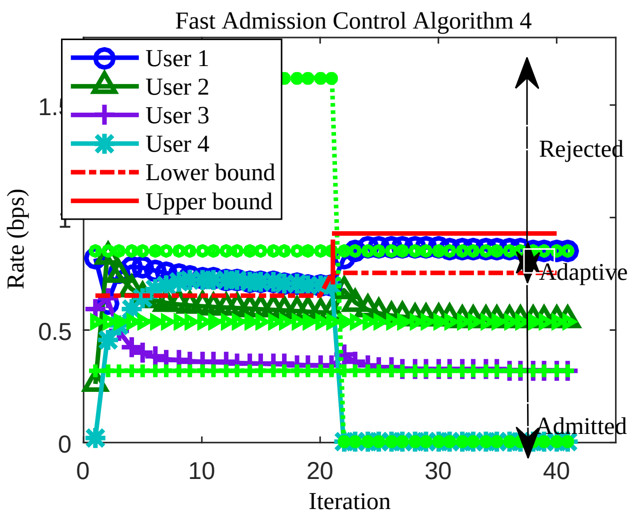


Fig. 7. An illustration of the convergence of Algorithm 4 for fast admission control in terms of individual rate when (10) is infeasible. The red solid line (i.e., Upper bound) is the optimal value obtained by Algorithm 1 for the network fairness with only the admitted Users 2 and 3. The red dashed line (i.e., Lower bound) is changed from the optimal value obtained by Algorithm 1 with all users to the optimal value obtained by Algorithm 1 with the admitted Users 2 and 3, and the adaptive User 1. Each green dashed line represents the corresponding demanded transmission rate for each user.

and Users 1, 3 and 4 communicate on the adaptive rate 0.628 Mbps as they are above the fairness obtained by Algorithm 1.

Example 4: It is interesting to note that the users' rate requirements which are above the lower bound of the network fairness may also be satisfied as shown in Figure 5 (b). Intuitively, we use the admission control to reject the users whose rate demands are beyond the upper bound of the network fairness as shown in Figure 2. Let the required rate vector be $\bar{\mathbf{r}} = [0.85, 0.54, 0.32, 1.62]$ which leads to the

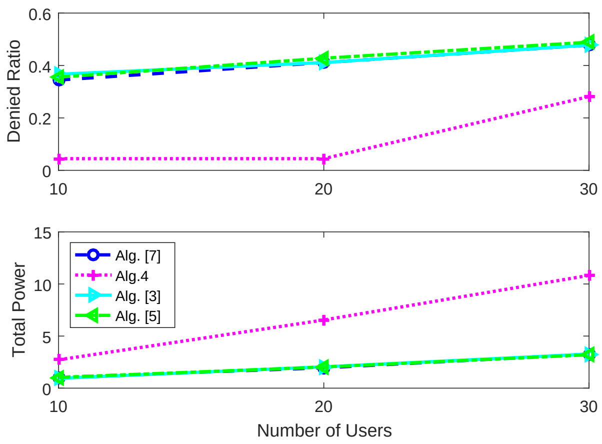


Fig. 8. Average outage probability and average total power consumption versus total number of users based on the Monte-Carlo simulations.

infeasible (10), and the channel gain is given as:

$$\mathbf{G} = \begin{bmatrix} 0.3183 & 0.1744 & 0.1500 & 0.1480 \\ 0.1905 & 0.6659 & 0.1618 & 0.1859 \\ 0.1805 & 0.1577 & 0.4098 & 0.1240 \\ 0.1887 & 0.1029 & 0.1490 & 0.4008 \end{bmatrix}. \quad (23)$$

Figure 7 shows the convergence of Algorithm 4. Users 2 and 3 are admitted because their rate demands are below the lower bound of the fairness 0.754 obtained by Algorithm 1 for Users 1, 2, and 3. User 4 is rejected because its rate demand is larger than the upper bound of the fairness 0.93 obtained Algorithm 1 for Users 2, and 3. User 1 is the adaptive user because its rate demand is between the lower and upper bounds of the fairness, thus its rate demand may be satisfied as Users 2 and 3 only achieve their rate demands instead of the lower bound of the fairness. Otherwise, the adaptive users should adapt their rates to the final lower bound of the network fairness.

Example 5: Compared to other approaches for larger wireless networks, this example reports the Monte-Carlo (MC) average results for 30 MC runs. For each MC run, transmitter locations are randomly drawn on a 2Km $\times$ 2Km square. For each transmitter location, a receiver location is drawn uniformly in a disc of radius 400 meters, excluding a radius of 10 meters. The rate demands of all users follow the uniform distribution on $[0.1, 0.9]$Mbps.

In Figure 8, Alg. 4 is our proposed Algorithm 4 in Section V, Alg. [3] is the centralized removal algorithm in [3], Alg. [7] is the distributed removal algorithm in [7] and Alg. [5] is the heuristic removal algorithm in [5]. Figure 8 shows that Algorithm 4 outperforms all the other approaches in terms of denied ratio at the cost of more power consumption. Because the global network interferences become higher due to the fact that our algorithm supports many adaptive users besides of the admitted users. Figure 9 demonstrates one example for thirty classified users, i.e., admitted, adaptive, and rejected users. Regardless of the delay from the transmitting information, Algorithm 4 only needs a few milliseconds within average fifty iterations in total. Note that Algorithm 4 is a double

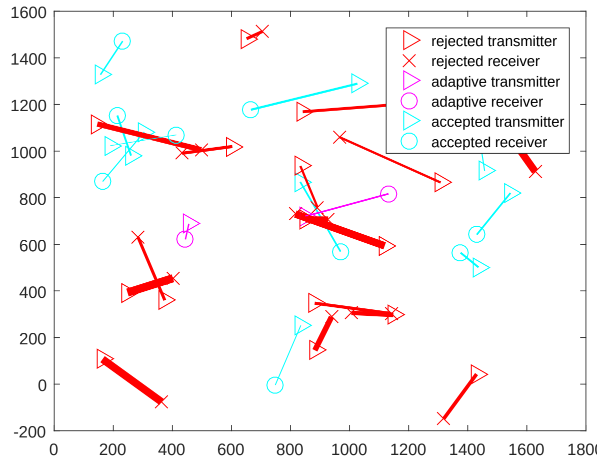


Fig. 9. An illustration of thirty classified users (source-destination pairs). Red, blue and purple lines denote the rejected, admitted and adaptive users, respectively. The width of line denotes the corresponding rate requirement.

layer circulation. While the inner circulation geometrically converges to the lower and upper bounds of the fairness, the outer circulation of classification is only in single digits for even one hundred users. In practice, the adaptive users have rights to decide whether to change channel for higher quality of service or to undertake the reduced rate for the fairness.

VIII. Conclusion

In this paper, we studied the novel data rate function based on the Signal to Interference Noise Ratio (SINR) and Q-function directly (i.e., $Q(\sqrt{\mathsf{SINR}})$), since the information about BER is essential critical to improve the correctness and performance of wireless networks. In order to guarantee the fairness for all users, we analyzed the optimal characteristics of the max-min rate fairness problem, and proposed a decentralized rate control algorithm. Then, we made use of the lower and upper bounds of the network fairness to address the feasibility issue of a total power minimization problem with rate constraints. We proposed a dynamic algorithm that adapted their rate demands to minimize the total power in a heterogeneous wireless network. Furthermore, we classified the users into three categories, i.e., *admitted users*, *adaptive users* and *rejected users*, so as to deal with more than one user at each time. Numerical evaluations demonstrated that our proposed algorithms exhibited faster convergence behavior.

Appendix

A. Proof of Lemma 1

Let $\mathbf{p}^\star$ be an optimal solution to (6). Suppose $\mathbf{A}\mathbf{p}^\star < \bar{\mathbf{p}}$. Now, let $\hat{\mathbf{p}}^\star = \left(\min_l \frac{\bar{p}_l}{(\mathbf{A}\mathbf{p}^\star)_l}\right)\mathbf{p}^\star$. Clearly, $\hat{\mathbf{p}}^\star$ is a feasible power allocation and $\hat{\mathbf{p}}^\star > \mathbf{p}^\star$. Moreover, since $\mathsf{SINR}(\alpha\mathbf{p}^\star)$ is strictly increasing in terms of α, we have that $\mathsf{SINR}_l(\hat{\mathbf{p}}^\star) > \mathsf{SINR}_l(\mathbf{p}^\star)$ for all l. For the rate function (2), we have:

$$\frac{\partial f_l(\mathsf{SINR}_l(\mathbf{p}))}{\partial \mathsf{SINR}_l(\mathbf{p})} = \frac{R}{\sqrt{2\pi\mathsf{SINR}_l(\mathbf{p})}} e^{-\mathsf{SINR}_l(\mathbf{p})/2} > 0, \quad (24)$$

because it is known about the first derivative of Q-function that:

$$\nabla Q(x) = -\frac{1}{\sqrt{2\pi}} e^{-x^2/2}. \quad (25)$$

It follows $f_l(\mathsf{SINR}_l(\hat{\mathbf{p}}^\star)) > f_l(\mathsf{SINR}_l(\mathbf{p}^\star))$ for all l, which contradicts the optimality of $\mathbf{p}^\star$.

B. Proof of Lemma 2

Let $\tau^\star$ be the optimal value of (6). Suppose not at optimality and the link l is with the largest weighted link rate, i.e.:

$$\frac{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right)}{\beta_l} > \tau^\star. \quad (26)$$

Recall that $G_{lj} > 0$ for all l and j. It is easy to verify that the $\mathsf{SINR}_l(\mathbf{p})$ is a strictly increasing function in p_l, and is a strictly decreasing function in p_j for all $j \neq l$. It is possible to decrease $p_l^\star$ by a sufficiently small amount $\epsilon > 0$ such that (26) is still satisfied by using the new transmit power for link l, i.e., $p_l = p_l^\star - \epsilon$. By doing so, $\min_j \frac{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_j(\mathbf{p})}\right)\right)}{\beta_j}$ including other links can increase. Thus, we can further increase the value of $\tau^\star$ which is a contradiction to the assumption of the optimality.

C. Proof of Theorem 1

From Lemma 2, we have that $(\tau^\star, \mathbf{p}^\star)$ must necessarily satisfy the following condition:

$$\tau^\star = \frac{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}^\star)}\right)\right)}{\beta_l}, \quad l = 1, \ldots, L. \quad (27)$$

Combining with Lemma 1, we cast (6) into the following eigenvalue problem:

$$\begin{cases} \frac{1}{\tau^\star} p_l^\star = \frac{\beta_l}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p}^\star)}\right)\right)} p_l^\star \\ \mathbf{A}\mathbf{p}^\star \leq \bar{\mathbf{p}}, \end{cases} \quad (28)$$

and $\tau^\star$ is maximal. Note that (28) is necessary but not sufficient for optimality unless the resulting $\tau^\star$ is maximal. However, we can show that there is a unique solution to (28) under given condition so that this necessary condition is also sufficient based on the nonlinear Perron-Frobenius theorem [27], [28]:

Let $\|\cdot\|$ be a monotone vector norm[1] on $\mathbb{R}^L$ and $\mathbb{S} = \{\mathbf{x} \in \mathbb{R}^L_{\geq 0} : \|\mathbf{x}\| = 1\}$. Let $\mathbf{f} : \mathbb{R}^L_{\geq 0} \to \mathbb{R}^L_{\geq 0}$ be a concave map[2] with $\mathbf{f}(\mathbf{x}) > \mathbf{0}$ for $\mathbf{x} \gneqq \mathbf{0}$[3]. Then $\mathbf{f}$ has a unique eigenvector $\mathbf{x}^\star \in \mathbb{S}$. Furthermore, $\mathbf{x}^\star$ is the unique fixed point of the normalized map $\mathbf{u} : \mathbb{S} \to \mathbb{S}$ defined by $\mathbf{u}(\mathbf{x}) = \frac{\mathbf{f}(\mathbf{x})}{\|\mathbf{f}(\mathbf{x})\|}$ and all the orbits of $\mathbf{u}$ converge geometrically fast to $\mathbf{x}^\star$ from any initial point $\mathbf{x}(0) \gneqq \mathbf{0}$.

[1]The vector norm $\|\cdot\|$ is monotone means $\|\mathbf{x}\| \geq \|\mathbf{y}\|$ if $\mathbf{x} \geq \mathbf{y} \geq \mathbf{0}$.

[2]A mapping $\mathbf{f} : \mathbb{R}^L_{\geq 0} \to \mathbb{R}^L_{\geq 0}$ is concave if $\mathbf{f}(\alpha\mathbf{x} + (1-\alpha)\mathbf{y}) \geq \alpha\mathbf{f}(\mathbf{x}) + (1-\alpha)\mathbf{f}(\mathbf{y})$ for all $0 \leq \alpha \leq 1$.

[3]$\mathbf{x} \gneqq \mathbf{y}$ implies $x_l \geq y_l$ for $1 \leq l \leq L$ and $\mathbf{x} \neq \mathbf{y}$.

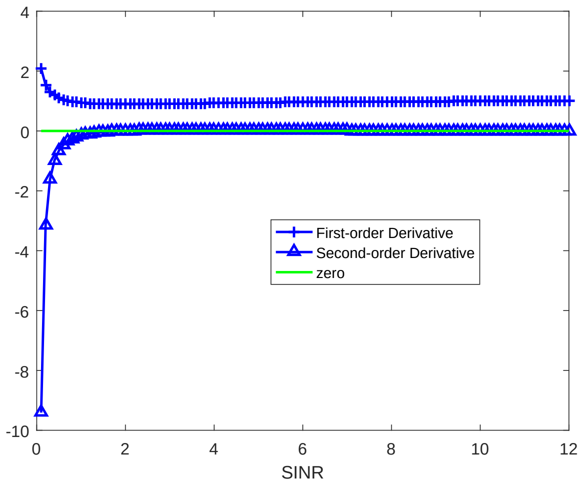


Fig. 10. The illustration of the corresponding first-order and second-order derivative for the function of $g(\mathsf{SINR}_l(\mathbf{p}))$ in terms of $\mathsf{SINR}_l(\mathbf{p})$.

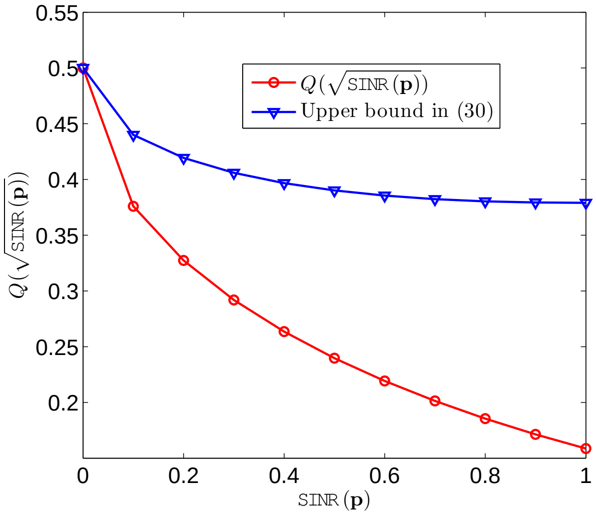


Fig. 11. The upper bound on the Q-function (3) as given by (37).

We next show that:

$$\begin{aligned} u_l(\mathbf{p}) &= \frac{\beta_l}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right)}p_l \\ &= \frac{\beta_l \mathsf{SINR}_l(\mathbf{p})}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right)}\left(\sum_{j\neq l} G_{lj}p_j+\sigma_l\right), \end{aligned} \tag{29}$$

is concave on the positive domain, which implies that (28) has a unique solution which can be found using a fixed-point iteration (c.f. Lemma 5 in [29]). To show that $u_l(\mathbf{p})$ is concave, we need the result from (c.f. Lemma 4 in [29] based on [30]), followed from the fact that the perspective and the affine composition of a function preserves the concavity of the function. Letting:

$$g(\mathsf{SINR}_l(\mathbf{p})) = \frac{\beta_l}{R(1-2Q(\sqrt{\mathsf{SINR}_l(\mathbf{p})}))}\mathsf{SINR}_l(\mathbf{p}). \tag{30}$$

Since:

$$\nabla g(x) = \frac{\beta_l}{R} \times \frac{1-2Q(\sqrt{x})-\sqrt{\frac{x}{2\pi}}e^{-x/2}}{(1-2Q(\sqrt{x}))^2}, \tag{31}$$

and:

$$\nabla^2 g(x) = \frac{\beta_l}{R}e^{-x/2} \times \frac{\frac{x-3}{2}(1-2Q(\sqrt{x}))+\sqrt{\frac{2x}{\pi}}e^{-x/2}}{(1-2Q(\sqrt{x}))^3\sqrt{2\pi x}}, \tag{32}$$

for $x>0$, therefore:

$$\nabla^2 g(x)\mid_{x=\mathsf{SINR}_l(\mathbf{p})} = \frac{\beta_l}{R}e^{-\mathsf{SINR}_l(\mathbf{p})/2} \times \frac{\frac{\mathsf{SINR}_l(\mathbf{p})-3}{2}(1-2Q(\sqrt{\mathsf{SINR}_l(\mathbf{p})}))+\sqrt{\frac{2\mathsf{SINR}_l(\mathbf{p})}{\pi}}e^{-\mathsf{SINR}_l(\mathbf{p})/2}}{(1-2Q(\sqrt{\mathsf{SINR}_l(\mathbf{p})}))^3\sqrt{2\pi\mathsf{SINR}_l(\mathbf{p})}}. \tag{33}$$

It can be seen that (30) is concave when the value of $\mathsf{SINR}_l(\mathbf{p})$ is below the threshold of near two, and nondecreasing which is shown in Figure 10. Even though the second-order derivative value is positive for large SINR, it is still below a certain small positive value ϵ. is concave as $\mathsf{SINR}_l(\mathbf{p})$ is concave and $g(\mathsf{SINR}_l(\mathbf{p}))$ is concave and nondecreasing.

Since $g(\mathsf{SINR}_l(\mathbf{p}))$ is concave, it follows that $u_l(\mathbf{p})$ is concave. Hence, we use the normalized fixed-point iteration to compute $\mathbf{p}^\star$:

$$\mathbf{p}(k+1) = \frac{\mathbf{u}(\mathbf{p}(k))}{\|\mathbf{u}(\mathbf{p}(k))\|^{\mathbf{A}}_{\infty,\bar{\mathbf{p}}}}, \tag{34}$$

which converges geometrically fast to $\mathbf{p}^\star$ for any $\mathbf{p}(0) > \mathbf{0}$. This iteration can be rewritten as Steps 2 and 3 of Algorithm 1. Note that Step 3 always gives a feasible power allocation.

D. *Proof of Theorem 2*

The burden of proof lies in the standard interference function framework of [13]. Thus, we first introduce it before showing how it is applied.

Definition 1: An interference function $\mathbf{I}(\mathbf{p})$ is standard if, for all $\mathbf{p} \geq 0$, the following properties are satisfied[4]:

- Monotonicity: If $\mathbf{p}_1 \geq \mathbf{p}_2$, then $\mathbf{I}(\mathbf{p}_1) \geq \mathbf{I}(\mathbf{p}_2)$.
- Scalability: For all $\alpha > 1, \alpha\mathbf{I}(\mathbf{p}) > \mathbf{I}(\alpha\mathbf{p})$.

Lemma 3: If $\mathbf{p}$ is a feasible power vector, then $\mathbf{I}(\mathbf{p})$ is a monotone increasing sequence of feasible power vector in a fixed-point iteration that converges to the unique fixed point $\mathbf{p}^\star$ that satisfies:

$$\mathbf{p}^\star = \mathbf{I}(\mathbf{p}^\star). \tag{35}$$

Lemma 4: Let $D \subseteq \mathbb{R}^K$ and $f : D \to \mathbb{R}$ be such that for all $x \in D$, $\frac{\partial f}{\partial x_l}, l = 1, \ldots, L$ exist on D. Then f is monotonically increasing on D if and only if $\frac{\partial f}{\partial x_l} \geq 0,\ l = 1, \ldots, L$.

Lemma 5: Let:

$$I_l(\mathbf{p}) = \frac{\bar{r}_l}{R\left(1-2Q\left(\sqrt{\mathsf{SINR}_l(\mathbf{p})}\right)\right)}p_l. \tag{36}$$

[4]Notice that, even though $\mathbf{p} \geq 0$ was required in [13], the results hold equally for just $\mathbf{p} > 0$. Moreover, notice that positivity (i.e., $\mathbf{I}(\mathbf{p}) > 0$) was required explicitly in [13], but can actually be implied by monotonicity and scalability since the latter two yield $\alpha\mathbf{I}(\mathbf{p}) > \mathbf{I}(\alpha\mathbf{p}) \geq \mathbf{I}(\mathbf{p})$, for $\alpha > 1$.

The interference function $\mathbf{I}(\mathbf{p})$ is standard [13].

This means that $\mathbf{I}(\mathbf{p})$ satisfies the three properties of the standard interference function:

- Positivity: Since the transmit power $\mathbf{p}$ is positive and the Q-function (3) is known to be $Q(x) < 0.5$ for $x > 0$, we have $I_l(\mathbf{p})$ is positive.
- Scalability: We scale each user's power by $\alpha > 1$. Then, we have $\mathsf{SINR}_l(\alpha\mathbf{p}) > \mathsf{SINR}_l(\mathbf{p})$. Using the chain rule of differentiation, we have $\partial f_l(\mathsf{SINR}_l(\mathbf{p}))/\partial\mathsf{SINR}_l(\mathbf{p}) > 0$. Thus, we have $f_l(\mathsf{SINR}_l(\alpha\mathbf{p})) \geq f_l(\mathsf{SINR}_l(\mathbf{p}))$. Then, we get $I_l(\alpha\mathbf{p}) < \alpha I_l(\mathbf{p})$ because of the positivity.
- Monotonicity: From Lemma 4, if we can verify that $\frac{\partial I_l(\mathbf{p})}{\partial p_j} \geq 0$ for all j, then the monotonicity of $I_l(\mathbf{p})$ is guaranteed. Furthermore, the monotonicity of $\mathbf{I}(\mathbf{p})$ is guaranteed.
  - If $j = l$, we have $\partial I_l(\mathbf{p})/\partial p_l \geq 0$ which is always satisfied by being equivalent to:

$$\begin{aligned} & Q(\sqrt{\mathsf{SINR}_l(\mathbf{p})}) \\ \leq \quad & \frac{1}{2} - \frac{\mathsf{SINR}_l^m(\mathbf{p}^m)}{2\sqrt{2\pi\mathsf{SINR}_l^m(\mathbf{p}^m)}} e^{-\mathsf{SINR}_l^m(\mathbf{p}^m)/2}, \end{aligned} \tag{37}$$

    as shown in Figure 11.
  - If $j \neq l$, similarly, we have $\partial I_l^m(\mathbf{p}^m)/\partial p_j^m \geq 0$.

Furthermore, the function $\mathbf{I}(\mathbf{p}) = \min\{\mathbf{I}(\mathbf{p}), \bar{\mathbf{p}}\}$ is still standard [13]. Therefore, the convergence of Algorithm 2 is guaranteed.

E. Proof of Corollary 1

Theorem 1 proves the convergence of $\mathbf{z}(k)$ in Steps 1 and 2 of Algorithm 3. Theorem 2 proves the convergence of $\mathbf{p}(k)$ in Step 3 of Algorithm 3.

F. Proof of Theorem 3

From (1) and (24), it is easy to verify that the l-th users' rate function (2) is a strictly increasing function in p_l, and is a strictly decreasing function in p_j for $j \neq l$. We can reduce the power p_l to be zero, which is regarded as being rejected from the networks. By doing so, all the other users' rates are increased while the l-th user's rate reduces to be zero. In other words, the interference in the networks is reduced when the users are rejected. In particular, the rejected users with zero rates are not taken into consideration when we obtain the optimal value of (5), i.e., the network fairness. Thus, the fairness obtained from Algorithm 1 will be larger after the users are rejected.

G. Proof of Corollary 2

Theorem 1 proves the convergence of $\mathbf{z}(k)$ in Step 2 and 3 of Algorithm 4. Theorem 2 and Corollary 1 prove the convergence of Algorithm 2 in Step 5 of Algorithm 4. The convergence of Step 4 is guaranteed by the limited number of users and Theorem 3.

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